A Model of Relative Thinking

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Abstract

Fixed differences loom smaller when compared to large differences. We propose a model of relative thinking where a person weighs a given change along a consumption dimension by less when it is compared to bigger changes along that dimension. In deterministic settings, the model predicts context effects such as the attraction effect, but predicts meaningful bounds on such effects driven by the intrinsic utility for the choices. In risky environments, a person is less likely to sacrifice utility on one dimension to gain utility on another that is made riskier. For example, a person is less likely to exert effort for a fixed return if there is greater overall income uncertainty. We design and run experiments to test basic model predictions, and find support for these predictions.

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1 Introduction

Amounts appear smaller when compared to larger things than when compared to smaller things. Perceptually, an inch on a yardstick seems smaller than an inch on a ruler. A price cut of $10 feels smaller to a person when she is considering spending $1000 on a product than when she is considering spending $100. Seeing a very big and very small house decreases the perceived size difference between two mid-size houses, and reduces a shopper's willingness to pay for improvements in size among that pair.

Though pervasive, the intuition of "relative thinking" is channeled in various ways in the psychology and economics literature. We follow Parducci (1965) and study how a given absolute difference can seem big or small depending on the *range* of alternatives under consideration. We incorporate range-based relative thinking into a model of economic choice, related in approach to earlier research by Bordalo, Gennaioli, and Shleifer (2012, 2013) and Koszegi and Szeidl (2013), but starting from different assumptions. Our main assumption is that a person puts less weight on a consumption dimension when outcomes along that dimension exhibit greater variability in the choice set he faces. This is consistent with the examples above and experimental evidence by Mellers and Cooke (1994), Soltani, De Martino, and Camerer (2012), and others that trade-offs depend on attribute ranges, where the impact of a given attribute difference is larger when presented in a narrower range.[1] For example, if a person is considering a set of different possible jobs that range narrowly in salary between $108,000 and $113,000, a $5,000 salary differential likely looms larger than it would if the salaries ranged between $68,000-$153,000.

Throughout this paper, we explore the model's general features and highlight how our "range-based" approach speaks to a number of empirical phenomena. The model matches known context effects from psychology and marketing—for example, proportional thinking (Thaler 1980, Tversky and Kahneman 1981) and attraction effects (Huber, Payne, and Puto 1982)—while clarifying limits to such effects.[2] It also generates novel, unexplored economic implications. In the context of discretionary labor choices, the model says that a worker will choose to exert less effort for a fixed

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[1] A similar theme is emphasized in recent neuroscience. Models of normalization, such as the notion of "range-adaptation" in Padoa-Schioppa (2009) or "divisive normalization" in Louie, Grattan and Glimcher (2011), tend to relate both the logic of neural activity, and the empirical evidence (reviewed in Rangel and Clithero 2012) on the norming of "value signals", to the possible role of norming in simple choices. Fehr and Rangel (2011) argue that "the best and worst items receive the same decision value, regardless of their absolute attractiveness, and the decision value of intermediate items is given by their relative location in the scale." Insofar as these models of value coding in the brain translate into choice, they may provide backing to the ideas discussed in this paper.

[2] Huber, Payne, and Puto (1982) suggest a mechanism similar to ours to account for attraction effects and asymmetric dominance, though Huber and Puto (1983) argue that it is difficult to explain the evidence as resulting from "range effects" because the likelihood of choice reversals seems to be insensitive to the magnitude of the range induced by the position of the decoy. Wedell (1991) shows something similar, and Simonson and Tversky (1992) cite all this evidence as "rejecting" the range hypothesis. However, meta-analysis by Heath and Chaterjee (1995) suggest that range effects do in fact exist in this context, as have more recent studies, such as Soltani, De Martino, and Camerer (2012).

return when there is greater overall income uncertainty. The more uncertain a worker is about daily earnings—realizing it could be very high or very low—the less motivated he will likely be to put in effort to earn an extra $10. We also designed experiments to confirm earlier experimental results that motivated our model, and to remove some of the confounds of those experiments. Results from every experiment we ran are supportive of wider ranges reducing people's sensitivity to differences within the range.

We present our model for deterministic environments in Section 2. A person's "consumption utility" for a K-dimensional consumption bundle c is separable across dimensions: $U(c) = \sum_k u_k(c_k)$. Rather than maximizing $U(c)$, however, we assume that a person instead makes choices according to "normed consumption utility" that depends not only on the consumption bundle c but also the comparison set C—which in applications we will equate with the choice set. We build on a recent economic literature begun by Bordalo, Gennaioli, and Shleifer (2012) in assuming that the comparison set influences choice through distorting the relative weights a person puts on consumption dimensions. Normed consumption utility equals $U^N(c|C) = \sum_k w_k \cdot u_k(c_k)$, where w_k captures the weight that the person places on consumption dimension k given the (notationally suppressed) comparison set C. Following Koszegi and Szeidl (2013), the weights $w_k > 0$ are assumed to be a function $w_k \equiv w(\Delta_k(C))$, where $\Delta_k(C) = \max_{\tilde{c} \in C} u_k(\tilde{c}_k) - \min_{\tilde{c} \in C} u_k(\tilde{c}_k)$ denotes the range of consumption utility along dimension k. Our key assumption, which departs from Koszegi and Szeidl (2013), is that $w(\Delta)$ is decreasing: the wider the range of consumption utility on some dimension, the less a person cares about a fixed utility difference on that dimension. For example, in searching for flights on a flight aggregator like Orbitz, our model says that spending extra money for convenience will feel bigger when the range of prices is $400 - $450 than when the range is $200 - $800. We also assume that $w(\Delta) \cdot \Delta$ is increasing, so that differences in normed utility are increasing in absolute magnitude when fixed as a proportion of the range: The $600 difference seems bigger when the range is $600 than the $50 difference seems when the range is $50. A basic implication of the model is a form of proportional thinking. For example—and consistent with examples and evidence by Savage (1954), Thaler (1980), Tversky and Kahneman (1981), and Azar (2011)—a person's willingness to exert effort to save money on a purchase is greater when the *relative* amount of money saved, measured in proportion to the range of spending under consideration, is higher.

The notion of proportional thinking that is inherent in range-based relative thinking is a frequent motivator for the idea that people exhibit diminishing sensitivity to changes the further those changes are from a reference point. Range-based relative thinking is different: In the presence of greater ranges along a dimension, our model says *all* changes along that dimension loom smaller. When considering possibilities of large-scale decisions, smaller stakes seem like peanuts. Figure 1 provides a visual illustration of the difference. According to range-based relative thinking, the space between the two dots looms smaller in the last three horizontal lines than in the first, visu-

ally representing greater ranges with longer lines. According to diminishing sensitivity, this space looms progressively smaller across the bottom three lines, visually representing the distance from the reference point as the distance from the left-most part of the line.

Figure 1: Visual Illustration of Diminishing Sensitivity vs. Relative Thinking
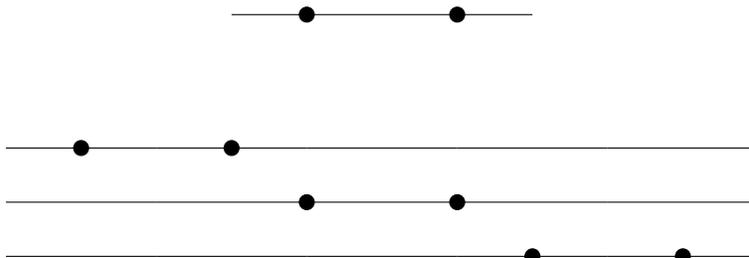
Section 3 explores context effects induced by relative thinking in riskless choice. When a person is indifferent between two 2-dimensional alternatives, the addition of a third to the comparison set influences his choices in ways consistent with experimental evidence. For example, our model predicts the "asymmetric dominance effect" proposed by Huber, Payne and Puto (1982): Adding a more extreme, inferior option to the choice set leads the person to prefer the closer of the two superior options. This is because the addition expands the range of the closer option's disadvantageous dimension by more than it expands the range of its advantageous dimension. Section 3 also characterizes limits of context effects based on bounds placed on the weighting function. Some inferior options, even if undominated, can never be "normed" into selection.

Section 4 extends the model to choice under uncertainty. The key assumption in this extension is that it is not only the range of expected values across lotteries that matters, but also the range of outcomes in the support of given lotteries. Roughly, we summarize each lottery's marginal distribution over $u_k(c_k)$ in terms of its mean plus or minus a measure of its variation. We then take the range along a dimension to equal the difference between the maximal mean-plus-variation among all lotteries vs. the minimal mean-minus-variation.

In Section 5, we spell out implications of a basic prediction of the Section 4 uncertainty model: People are more inclined to sacrifice on a dimension when it is riskier. As in the worker example above, people are less willing to put effort into making money when either a) they earn money simultaneously from another stochastic source, or b) they faced a wider range of *ex ante* possible returns. In both cases, the wider range in the monetary dimension due to uncertainty lowers the workers' sensitivity to incremental changes in money. Likewise, in these cases the wider range makes effort choices less sensitive to the level of monetary incentives. Finally, workers are less willing to put in effort for a fixed return when they expected the opportunity to earn more, because this also expands the range on the monetary dimension and makes the fixed return feel small.

Section 3 below briefly compares our approach to the predictions of Bordalo, Gennaioli, and Shleifer's (2012, 2013) approach to studying the role of salience in decision-making, and to more recent models by Kőszegi and Szeidl (2013) and Cunningham (2013). Appendix C shows in more detail how and when our model generates substantively different predictions to these. In particular, none universally share our property that given differences along a dimension loom smaller in the presence of bigger ranges.[3]

We designed several experiments to explore this basic property. In Section 6, we present these experiments, which confirm earlier experimental results that motivated our model while eliminating some of the confounds of those earlier experiments. We first present results from several vignette studies along the lines of classic proportional-thinking vignettes, but designed to isolate the impact of range effects. For example, we ask one group of participants to imagine that they went to a store to buy a phone, expecting to pay between $490 and $510 and expecting the trip to the store to last between 5 and 55 minutes. We ask another group of participants to instead imagine that they went to a store to buy a phone, expecting to pay between $190 and $810 and expecting the trip to the store to last between 15 and 45 minutes. The first group faced an ex ante narrow range over money and wide range in time and the latter faced an ex ante wide range over money and narrow range in time. We then tell both groups of participants to imagine that they've spent 30 minutes at the store and it turns out that the phone they want costs exactly $500. Finally, we ask them whether they'd be willing to fill out a 15-minute survey to save $5 on the phone. The prediction of our model, which our results support, is that participants in the first group will be more willing to complete the survey than participants in the latter group.

We also present results from a real-effort study that takes care to separate range effects from the impact of modifying features of comparison sets, such as averages, that play a major role in Bordalo, Gennaioli, and Shleifer's (2013) and Cunningham's (2013) theories. Specifically, we incentivize participants to accurately report their preference ranking over four combinations of tasks and money, where they are trading off receiving more money for needing to do more tasks. One group of participants faced a menu of options with a wide range of the amount of tasks to complete and a narrow range of money; the other faced a menu with a narrow range of the amount of tasks to complete and a wide range of money. For reasons we describe in Section 6, we designed the menus such that the best and worst options were obvious and our interest was in which options participants ranked second versus third. These middle options were shared across menus, with the

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[3]Azar (2007) provides a theory of relative thinking built on diminishing sensitivity, where people are less sensitive to price changes at higher price levels. Contemporaneously, Kontek and Lewandowski (2017) present a model of range-dependent utility to study risk preferences in choices over single-dimensional lotteries. They assume that the outcomes of a given lottery are normed only according to the range of outcomes within the support of that lottery—other lotteries in the choice set do not influence a lottery's normed utility. Thus, unlike our model, theirs is not one of context-dependent choice or valuation.

prediction that participants in the wide range of tasks and narrow range of money treatment would be more likely to rank the money-advantaged middle option above the fewer-tasks-advantaged middle option. Results from this and every experiment we ran are supportive of wider ranges reducing people's sensitivity to fixed differences within the range.

In many ways, the framework for our riskless model most closely resembles Kőszegi and Szeidl's (2013) model of focusing, and indeed elements of our formalism build directly from it.[4] But, in reduced-form, we say the range in a dimension has the opposite effect as it does in their model. Nevertheless, the basic force underlying their approach, as well as that of Bordalo, Gennaioli, and Shleifer (2012, 2013)—that attentional and focusing issues can lead wider ranges to enhance the weight a person places on a dimension—is both compelling and not inherently opposed to range-based relative thinking. Intuitively, focusing effects work through differences in the amount of attention paid to different dimensions; relative thinking effects through differences in the amount of attention paid to *changes* along different dimensions. To sharpen this point, we sketch a framework for studying the interaction between focusing effects and relative thinking in Section 7. This framework reflects an intuition that relative thinking effects could dominate focusing effects in many of the two-dimensional choice situations that we focus on in this paper, including in our experimental tests.

We conclude in Section 8 by discussing omissions and shortcomings of our model.

2   Relative Thinking: The Deterministic Case

We begin by presenting a special case of the model that applies to situations where a person chooses from sets of riskless options. In later sections we present the full model which also enables us to study choice under uncertainty, defining ranges as a function of available lotteries. The agent's "consumption" or "un-normed" utility for a riskless outcome is $U(c) = \sum_k u_k(c_k)$, where $c = (c_1, \ldots, c_K) \in \mathbb{R}^K$ is consumption and we assume each $u_k(c_k)$ is strictly increasing in c_k. However, the person does not maximize consumption utility, but rather "normed" utility, which is denoted by $U^N(c|C)$ given "comparison set" C.[5]

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[4]Kőszegi and Szeidl's (2013) model does not study uncertainty; extending their model along the lines of our extension in Section 4 would presumably be straightforward. The underlying psychology of our model is more closely related to Cunningham (2013). Cunningham (2013) models proportional thinking in relation to the average size of attributes rather than as a percentage of the range, making his predictions dependent on the choice of a reference point against which the size of options is defined. In our understanding of Cunningham (2013) as positing zero as the reference point, a person would be *less* sensitive to paying $1200 rather than $1100 for convenience if the choices ranged between $1100 and $1200 than if they ranged between $400 and $1300. Our model says the narrower range would make people more sensitive; models of diminishing sensitivity would say it would not matter (fixing the reference point).

[5]Although we do not emphasize normative implications through most of the paper, our perspective is that norming may influence choice without affecting experienced utility.

Throughout this paper, we equate the comparison set with the (possibly stochastic) choice set, though the model setup is more general.[6] Our model assumes that the comparison set influences choice by distorting the relative weight a person puts on each consumption dimension. More specifically, normed consumption utility equals

$$U^N(c|C) = \sum_k u_k^N(c_k|C) = \sum_k w_k \cdot u_k(c_k),$$

where w_k captures the weight that the decision-maker places on consumption dimension k given comparison set C.[7]

We make the following assumptions on w_k:

Norming Assumptions in the Deterministic Case:

> *N0(d)*. The weights w_k are given by $w_k = w(\Delta_k(C))$, where $\Delta_k(C) = \max_{c \in C} u_k(c_k) - \min_{c \in C} u_k(c_k)$ denotes the range of consumption utility along dimension k.
>
> *N1*. $w(\Delta)$ is a differentiable, decreasing function on $(0, \infty)$.
>
> *N2*. $w(\Delta) \cdot \Delta$ is defined on $[0, \infty)$ and is strictly increasing.

The first two assumptions capture the psychology of relative thinking, motivated by evidence and discussions in Parducci (1965), Mellers and Cooke (1994) and Soltani, De Martino, and Camerer (2012): the decision-maker attaches less weight to a given change along a dimension when the range of consumption utility along that dimension is higher. Put differently, as confirmed by Proposition 1 below, these assumptions imply that a particular advantage or disadvantage of one option relative to another looms larger when it represents a greater percentage of the overall range.[8] We provide further support for this basic prediction in Section 6 below.

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[6] When a person makes plans prior to knowing which choice set he will face, options outside the *realized* choice set can matter. We will more carefully describe how this works when we consider choices over lotteries in Section 4.

[7] While this formulation is sufficient for analyzing choice behavior in the way we do, as we discuss in Section 8 this formulation could be inadequate to do cross-choice-set welfare analysis. In that context, we could instead consider mathematically re-normalized formulations such as

$$\tilde{U}^N(c|C) = \sum_k \min_{\tilde{c} \in C} u_k(\tilde{c}_k) + w_k \cdot (u_k(c_k) - \min_{\tilde{c} \in C} u_k(\tilde{c}_k)).$$

This re-normalization to $\min_{\tilde{c} \in C} u_k(\tilde{c}_k)$ may provide a more natural interpretation across contexts, since it implies that normed and un-normed decision utility coincide given singleton comparison sets.

[8] Even if one somehow found natural units to choose within a dimension, the natural unit of comparison is utility rather than consumption levels given our interest in tradeoffs across dimensions. In terms of capturing the psychophysics, using utility may miss neglect of diminishing marginal utility: a person faced with 100 scoops of ice cream may treat the difference between 2-3 scoops as smaller than if he faced the possibility of getting 5 scoops, even though he may be satiated at 5 scoops.

Although one could use notions of dispersion that rely on more than just the endpoints, we follow Parducci (1965), Mellers and Cooke (1994), and Kőszegi and Szeidl (2013) by assuming that the range of consumption utility in the deterministic case (the "d" in *N0(d)* stands for deterministic) is simply the difference between the maximum value and the minimum value. Of course, at some level this is an approximation: levels of dispersion besides the literal range almost surely matter for choices.[9] We present the full assumption *N0* that handles lotteries below in Section 4.

Assumption *N2* assures that people are sensitive to absolute consumption utility differences. If a person likes apples more than oranges, then he strictly prefers choosing an apple when the comparison set equals {(1 apple, 0 oranges), (0 apples, 1 orange)}. While *N1* says that the decision weight on the "apple dimension" is lower than the decision weight on the "orange dimension"— since the range of consumption utility on the apple dimension is higher—*N2* guarantees that the trade-off between the two dimensions still strictly favors picking the apple. In particular, giving up 100% the range on the apple dimension looms strictly larger than gaining 100% the range on the orange dimension. This assumption is equivalent to assuming that the decision weight is not too elastic with respect to the range. In the limiting case where $w(\Delta) \cdot \Delta$ is constant in Δ, the agent only considers percentage differences when making decisions.

We sometimes add a final assumption, which bounds the impact of relative thinking:

N3. $\lim_{\Delta \to \infty} w(\Delta) \equiv w(\infty) > 0$ and $w(0) < \infty$.

This assumption says that a given difference in consumption utility is never negated by norming, and arbitrarily large differences are arbitrarily large even when normed. Similarly, it says a given difference in consumption utility is never infinitely inflated by norming, and arbitrarily small differences are arbitrarily small even when normed. While we assume that *N0-N2* hold throughout the paper, we specifically highlight when results rely on *N3*, since the limiting behavior of $w(\cdot)$ only matters for a subset of the results and we view this assumption as more tentative than the others.

The notation and presentation implicitly build in an important assumption: The weight on a dimension depends solely on the utility range in that dimension. The $w(\Delta)$ function yields sharp predictions once a cardinal specification of utilities is chosen—with the important restriction to

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[9]In fact, research in psychophysics following Parducci's (1965) "range-frequency theory" documents the importance of both the range effects we emphasize, as well as frequency effects: An outcome of fixed position within the range appears smaller when its percentile rank is lower among outcomes in the judgement context.

dimension-separable utility functions.[10] To take a parameterized example, consider

$$w(\Delta) = (1 - \rho) + \rho \frac{1}{\Delta + \xi},$$

where $\rho \in [0, 1)$ and $\xi \in (0, \infty)$. When $\rho = 0$, the model corresponds to the classical, non-relative-utility model, where a person only considers level differences when making trade-offs. When $\rho > 0$, a marginal change in underlying consumption utility looms smaller when the range is wider. Compared to the underlying utility, people act as if they care less about a dimension the wider the range of utility in that dimension. Note that Assumptions *N2* and *N3* hold: *N3* requires $\rho < 1$ and $\xi > 0$. In the limit case as $\rho \to 1$ and $\xi \to 0$, the actual utility change on a dimension from different choices does not matter, just the percentage change in utility on that dimension.

In Appendix B we discuss a method for determining both $u_k(\cdot)$ and $w(\cdot)$ from behavior, which closely follows the approach in Kőszegi and Szeidl (2013). The elicitation assumes that we know how features of options map into consumption dimensions and that we can separately manipulate individual dimensions. (We discuss below how it is sometimes possible to test whether a person treats two potentially distinct dimensions as part of the same or separate dimensions in situations where this is not obvious.) It also imposes the norming assumptions *N0* and *N2* — assumptions shared by Kőszegi and Szeidl (2013) — but not our main assumption *N1* that $w(\cdot)$ is decreasing: Indeed, the elicitation can be used to test our assumption against Kőszegi and Szeidl's (2013) that $w(\cdot)$ is increasing. The algorithm elicits consumption utility by examining how a person makes tradeoffs in "balanced choices", for example between $(0, 0, a, b, 0, 0)$ and $(0, 0, d, e, 0, 0)$, where assumptions *N0* and *N2* guarantee that the person will choose to maximize consumption utility. After consumption utilities have been elicited, the algorithm then elicits the weighting function $w(\cdot)$ by examining how ranges in consumption utility influence the rate at which the person trades off utils across dimensions. But, as spelled out below and taken to data in Section 6, the model makes directional comparative statics predictions in situations where $w(\cdot)$ is not fully recoverable.

The model implies a form of proportional thinking. For any two consumption vectors $c', c \in \mathbb{R}^K$, define $\delta(c', c) \in \mathbb{R}^K$ as a vector that encodes absolute utility differences between c' and c along different consumption dimensions: For all k,

$$\delta_k(c', c) = u_k(c'_k) - u_k(c_k).$$

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[10]Once $w(\cdot)$ is fixed, affine transformations of $U(\cdot)$ will not in general result in affine transformations of the normed utility function. As such, like other models that transform the underlying "consumption" utility function, either $U(\cdot)$ must be given a cardinal interpretation or the specification of $w(\cdot)$ must be sensitive to the scaling of consumption utility. Our formulation also assumes additive separability, though we could extend the model to allow for complementarities in consumption utility to influence behavior similarly to how Kőszegi and Szeidl (2013, footnote 7) suggest extending their focusing model.

Choice depends not only on these absolute differences, but also on proportional differences, $\delta_k(c',c)/\Delta_k(C)$. To highlight this, we will consider the impact of "widening" choice sets along particular dimensions. Formally:

Definition 1. $\tilde{C}$ is *a k-widening of C* if

$$\Delta_k(\tilde{C}) > \Delta_k(C)$$
$$\Delta_i(\tilde{C}) = \Delta_i(C) \text{ for all } i \neq k.$$

In words, $\tilde{C}$ is a k-widening of C if it has a greater range along dimension k and the same range on other dimensions. Although widening may connote set inclusion, our definition does not require this. In our model, the assessment of advantages and disadvantages depends on the range, not on the position within the range or on the position of the range with respect to a reference point.

Proposition 1. *Let* $C, \tilde{C} \subset \mathbb{R}^K$ *where* $\tilde{C}$ *is a k-widening of C.*

1. *Suppose the person is willing to choose c from C. Then for all* $\tilde{c} \in \tilde{C}, \tilde{c}' \in \tilde{C}$, *and* $c' \in C$ *such that*

$$\delta_k(\tilde{c}, \tilde{c}') > \delta_k(c, c') > 0$$
$$\frac{\delta_k(\tilde{c}, \tilde{c}')}{\Delta_k(\tilde{C})} = \frac{\delta_k(c, c')}{\Delta_k(C)}$$
$$\delta_i(\tilde{c}, \tilde{c}') = \delta_i(c, c') \text{ for all } i \neq k,$$

   the person will not choose $\tilde{c}'$ *from* $\tilde{C}$.

2. *Suppose the person is willing to choose c from C. Then for all* $\tilde{c} \in \tilde{C}, \tilde{c}' \in \tilde{C}$, *and* $c' \in C$ *such that*

$$\delta_k(c, c') < 0$$
$$\delta_i(\tilde{c}, \tilde{c}') = \delta_i(c, c') \text{ for all } i,$$

   the person will not choose $\tilde{c}'$ *from* $\tilde{C}$.

Part 1 of Proposition 1 says that a person's willingness to choose consumption vector c over consumption vector c' is increasing in the absolute advantages of c relative to c', fixing proportional advantages.[11] But Part 2 says the willingness to choose c is also increasing in its relative

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[11]We add the assumption that $\delta_k(\tilde{c}, \tilde{c}') > \delta_k(c, c') > 0$ for clarity, but this is implied by $\delta_k(\tilde{c}, \tilde{c}')/\Delta_k(\tilde{C}) = \delta_k(c, c')/\Delta_k(C)$ together with $\tilde{C}$ being a k-widening of C.

advantages, measured in proportion to the range. To illustrate, suppose each c is measured in utility units. Then if the person is willing to choose $c = (2,1,0)$ from $C = \{(2,1,0),(1,2,0)\}$, Part 1 says that he is not willing to choose $\tilde{c}' = (3,2,0)$ from $\tilde{C} = \{(6,1,0),(3,2,0)\}$, which has a bigger range on the first dimension. Part 2 further says that he is not willing to choose $\tilde{c}' = (4,5,3)$ from $\tilde{C} = \{(5,4,3),(4,5,3),(5,0,3)\}$, which has a bigger range on the second dimension.

To take a more concrete example, Proposition 1 implies that a person's willingness to exert e units of effort to save \$x on a purchase is greater when the relative amount of effort, measured in proportion to the range of effort under consideration, is lower or the relative amount of money saved, measured in proportion to the range of spending under consideration, is higher. To illustrate, consider the choice between consumption vectors (measured in utils)

$$
\begin{array}{lccc}
\text{don't purchase:} & 0 & 0 & 0 \\
\text{purchase without exerting effort to save \$:} & v & -p & 0 \\
\text{purchase and exert effort to save \$:} & v & -p+x & -e,
\end{array}
$$

where the first dimension is the utility from the product, the second the utility from money (p is the price), and the third the utility from not exerting effort. The ranges on the three dimensions are (v,p,e), so the person prefers exerting effort e to save \$x on a \$p purchase if and only if $w(e) \cdot e < w(p) \cdot x$. Since the right hand side of this expression is decreasing in p, the person is less willing to exert effort to save a fixed amount of money when the base price of the product is higher.

In this manner, the model is consistent with evidence used to motivate proportional thinking, such as Tversky and Kahneman's (1981) famous "jacket-calculator" example, based on examples by Savage (1954) and Thaler (1980) and explored further by Azar (2011)—where people are more willing to travel 20 minutes to save \$5 on a \$15 purchase than on a \$125 purchase—so long as not buying is an option.[12] Diminishing sensitivity could also explain this pattern, but relative thinking makes additional intuitive predictions. For example, relative thinking says that traveling 20 minutes to save \$5 on a purchase seems more attractive when it is also possible to travel 50

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[12]Our explanation requires that "don't buy" is in the comparison set. While not explicitly stated as an option in the original problem above, we conjecture that people nevertheless contemplate the possibility of not buying the jacket and calculator.

Like explanations based on diminishing sensitivity, ours also relies on the idea that people narrowly bracket spending on a given item. Indeed, Tversky and Kahneman (1981) fix total spending in their example—they compare responses across two groups given the following problem, where one group was shown the values in parantheses and the other was shown the values in brackets:

> Imagine that you are about to purchase a jacket for (\$125) [\$15], and a calculator for (\$15) [\$125]. The calculator salesman informs you that the calculator you wish to buy is on sale for (\$10) [\$120] at the other branch of the store, located 20 minutes drive away. Would you make the trip to the other store?

If people broadly bracketed spending on the two items, then the range of spending would be the same across the two groups, and our model would not be able to account for the difference in the propensity to make the trip.

minutes to save $11. When not buying at all is an option, the additional savings does not expand the range along the money dimension, but does expand the range of time costs and therefore makes traveling 20 minutes seem small.

Contrasting the following example with and without the brackets may also help build intuition: "Imagine you're about to buy a [$50] calculator for $25 [by using a 50% off coupon, which was unexpectedly handed to you when you entered the store]. A friend informs you that you can buy the same calculator for $20 at another store, located 20 minutes drive away. Would you make the trip to the other store?" Range-based relative thinking predicts that people would be less likely to make the trip in the bracketed condition. Diminishing sensitivity can explain this as well but only if we take the reference price to equal the expected price paid rather than zero. However, if we adopt the reference price to equal the expected price paid, diminishing sensitivity can no longer explain the original jacket/calculator pattern. See Section 6 for related experimental evidence.

Another property of the model is that the decision-maker's choices will be overly sensitive to the *number* of distinct advantages of one option over another, and insufficiently sensitive to their size. Consider the following consumption vectors (expressed in utils)

$$c^1 = (2,3,0)$$
$$c^2 = (0,0,5).$$

When $C = \{c^1, c^2\}$, then the decision-maker will exhibit a strict preference for c^1 over c^2 despite underlying consumption utility being equal: $2w(2) + 3w(3) > 5w(5)$, since $w(\cdot)$ is decreasing by assumption *N1*, implying that $2w(2) + 3w(3) > 5w(3) > 5w(5)$.

More starkly, consider a limiting case of assumption *N2* where $\Delta w(\Delta)$ is constant in Δ, so the decision-maker cares only about proportional advantages and disadvantages relative to the range of consumption utility. When two consumption vectors span the range of consumption utility, each advantage and disadvantage represents 100% of the range and looms equally large. Comparing the two vectors thus reduces to comparing the *number* of advantages and disadvantages in this case. Appendix A.1 develops a more general result on how, all else equal, relative thinking implies that the attractiveness of one consumption vector over another goes up when its advantages are spread over more dimensions or its disadvantages are more integrated. This appendix also discusses this result in light of evidence by Thaler (1985) and others on how people have a tendancy to prefer segregated gains and integrated losses, though the evidence on losses is viewed as far less robust.

The person's sensitivity to the distribution of advantages across dimensions means that, in cases where the dimensions are not obvious, it may be possible to test whether a person treats two potentially distinct dimensions as part of the same or separate consumption dimensions.[13] To

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[13]This feature of the model also means that choice behavior can exhibit cycles, so we have to be careful in how

adapt an example from Kőszegi and Szeidl (2013, Appendix B), suppose an analyst is uncertain whether a person treats a car radio as part of the same attribute dimension as a car. The analyst can test this question by finding: The price p that makes the person indifferent between buying and not buying the car; the additional price p' that makes the person indifferent between buying the car plus the car radio as opposed to just the car; and finally testing whether the person would buy the car plus the car radio at $p + p'$ dollars. If the person treats the car radio as part of the same attribute dimension as the car, then he will be indifferent. On the other hand, if he treats it as part of a separate dimension then he will not be indifferent; moreover, the direction of preference can identify whether $w(\Delta)$ is decreasing or increasing. Under our model, a person who treats the car radio as a separate attribute will strictly prefer buying the car plus the radio at $p + p'$ dollars because relative thinking implies that the person prefers segregated advantages.[14]

3 Contextual Thinking and Choice-Set Effects

3.1 Classical Choice-Set Effects

The relative-thinker's preference between two given alternatives depends on the totality of alternatives under consideration. To avoid cumbersome notation, assume each c is measured in utility units throughout this discussion. Consider a pair of 2-dimensional consumption vectors, c, c', which have the property that $U(c) = c_1 + c_2 = c'_1 + c'_2 = U(c')$, where $c'_1 > c_1$ and $c'_2 < c_2$. That is, moving from c to c' involves sacrificing some amount on the second dimension to gain some on the first. When c and c' are the only vectors under consideration, the relative-thinker is indifferent between them: $U^N(c'|\{c,c'\}) - U^N(c|\{c,c'\}) = w(c'_1 - c_1) \cdot (c'_1 - c_1) - w(c_2 - c'_2) \cdot (c_2 - c'_2) = 0$. What happens if we add a third consumption vector c''?

Figure 2 illustrates how the addition of an option influences the preference between c and c'. After presenting the full model that handles choice under uncertainty in Section 4, we will return to the figure and consider the area to the right of the diagonal, which depicts the impact of adding a superior option to the set. Focusing for now on inferior options, when c'' falls in the lighter

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we elicit preferences when people are relative thinkers. To take an example, suppose consumption is expressed in utils and consider $c = (2,2,0)$, $c' = (4,0,0)$, and $c^y = (0,0,y)$. A relative thinker expresses indifference between c and c' from a binary choice set (because this is a "balanced choice"), as well as between c' and $c^{y=4}$. But, because the person acts as if he prefers segregated advantages, he expresses a strict preference for c over $c^{y=4}$ from a binary choice set. Interpreting the third dimension as "money" and the first two dimensions as attributes, the person would express indifference between the attribute components of c and c' if asked to choose between them but appear to place a greater dollar value on c. Under the view that $U(\cdot)$ is the correct welfare metric, the binary choice elicitation yields the more accurate conclusion in this example.

[14]In Kőszegi and Szeidl's (2013) model, such a person will strictly prefer not to buy at this price because of their bias towards concentration. Likewise, diminishing sensitivity does not share this prediction with a reference point of zero, but rather implies that the person would be indifferent.

blue area in the bottom region, its addition expands the range on c''s disadvantageous dimension by more than it expands the range on its advantageous dimension and leads the relative-thinker to choose c' over c. Symmetrically, when c'' falls in the darker grey area in the left region, its addition expands the range on c''s advantageous dimension by more than it expands the range on its disadvantageous dimension and leads the relative-thinker to choose c over c'. Finally, when c'' falls in the white area in the middle region, its addition does not affect the range on either dimension and the relative-thinker remains indifferent between c and c'.

Figure 2: The impact of adding the *ex ante* possibility of being able to choose c'' on the relative-thinker's choice from realized choice-set $\{c, c'\}$, where each dimension is measured in utility units.
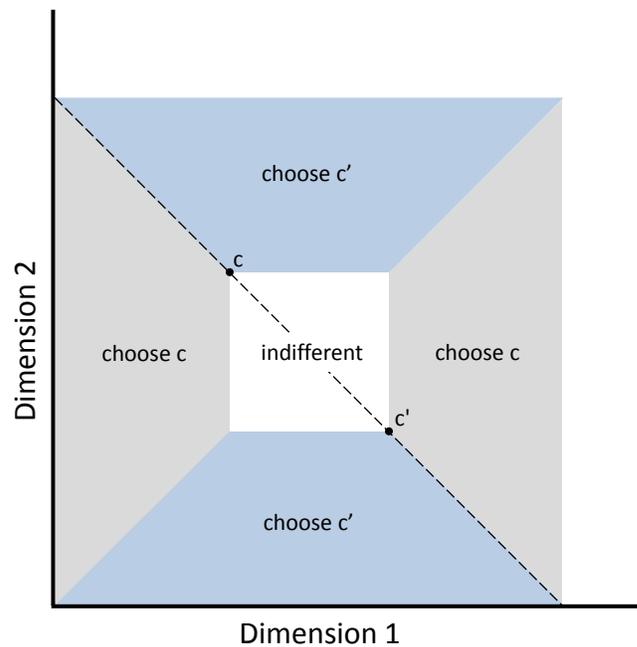
To illustrate, suppose a person is deciding between the following jobs:

Job X. Salary: 100K, Days Off: 199
Job Y. Salary: 110K, Days Off: 189
Job Z. Salary: 120K, Days Off: 119,

where his underlying utility is represented by $U = \text{Salary} + 1000 \times \text{Days Off}$. A relative thinker would be indifferent between jobs X and Y when choosing from $\{X, Y\}$, but instead strictly prefer the higher salary job Y when choosing from $\{X, Y, Z\}$: The addition of Z expands the range of

Y's disadvantage relative to *X*—days off— by more than it expands the range of *Y*'s advantage—salary.[15]

This pattern is consistent with the experimental evidence that adding an inferior "decoy" alternative to a choice set increases subjects' propensity to choose the "closer" of the two initial alternatives. Famously, experiments have found that adding an alternative that is dominated by one of the initial options, but not the other, increases the preference for the induced "asymmetrically dominant" alternative.[16] This effect, called the *asymmetric dominance effect* or *attraction effect* was initially shown by Huber, Payne and Puto (1982), and has been demonstrated when subjects trade off price vs. quality or multiple quality attributes of consumer items (e.g., Simonson 1989), the probability vs. magnitude of lottery gains (e.g., Soltani, De Martino, and Camerer 2012), and various other dimensions including demonstrations by Herne (1997) over hypothetical policy choices and Highhouse (1996) in hiring decisions.[17] Consistent with our model, similar effects (e.g., Huber and Puto 1983) are found when the decoy is not dominated but "relatively inferior" to one of the two initial alternatives. There is also evidence that context effects are more pronounced when the decoy is positioned "further" from the original set of alternatives, increasingly expanding the range of one of the dimensions (Heath and Chatterjee 1995; Soltani, De Martino, and Camerer 2012).

Our basic results on choice-set effects contrast with those of Bordalo, Gennaioli, and Shleifer (2013) and other recent models, including Kőszegi and Szeidl (2013) and Cunningham (2013), that likewise model such effects as arising from features of the choice context influencing how attributes of different options are weighed. We describe this in detail in Appendix C, and Somerville

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[15]Our basic results on choice-set effects also highlight a particular way in which the relative thinker expresses a taste for "deals" or "bargains". The addition of the "decoy" job *Z* makes Job *Y* look like a better deal in the above example—while getting 10K more salary in moving from Job *Y* to Job *Z* requires giving up 70 days off, getting 10K more salary in moving from Job *X* to Job *Y* only requires giving up 10 days off. But our model fails to capture some behavioral patterns that may reflect a taste for bargains. Jahedi (2011) finds that subjects are more likely to buy two units of a good at price p when they can get one for slightly less. For example, they are more likely to buy two apple pies for $1.00 when they can buy one for $.96. While a taste for bargains may undergird this pattern, our model does not predict it: adding one apple pie for $.96 to a choice set that includes not buying or buying two apple pies for $1.00 does not expand the range on either the money or the "apple pie" dimensions.

[16]We emphasize laboratory evidence on attraction effects because we believe it directly speaks to basic predictions of choice-set dependent models. For various reasons, some of which are emphasized below in Sections 3.2 and 8, we are less convinced that these effects are necessarily important in the field. Recent studies that provide other reasons to question the practical significance of attraction effects include Frederick, Lee, and Baskin (2014) as well as Yang and Lynn (2014).

[17]While initial demonstrations by Huber, Payne, and Puto (1982) and others involved hypothetical questionnaires, context effects like asymmetric dominance have been replicated involving real stakes (Simonson and Tversky 1992; Doyle, O'Connor, Reynold, and Bottomley 1999; Herne 1999; Soltani, De Martino, and Camerer 2012; Somerville 2019). They have also been demonstrated in paradigms where attempts are made to control for rational inference from contextual cues (Simonson and Tversky 1992, Prelec, Wernerfelt, and Zettelmeyer 1997, Jahedi 2011)—a potential mechanism fomalized by Wernerfelt (1995) and Kamenica (2008). Closely related is the "compromise effect" (Simonson 1989), or the finding that people tend to choose middle options.

(2019) as well as Landry and Webb (2019) provide more extensive treatments on differences between models.[18] Here, we primarily confine attention to comparing our predictions to Kőszegi and Szeidl's (2013) because their model also assumes that decision weights are solely a function of the range of consumption utility, which makes it the simplest to compare. Since it makes the opposite assumption on how the range matters, namely that decision weights are *increasing* in the range, it makes opposite predictions to ours in all two-dimensional examples along the lines illustrated in Figure 2. Their model says, for example, that adding Job Z will lead people to choose Job X because its addition draws attention to Days Off. The predictions of their model in two-dimensional examples seem hard to reconcile with the laboratory evidence on attraction effects summarized above, as well as with our experimental evidence presented in Section 6. We explore the possibility in Section 7 that "focusing effects" may be more important in choice problems involving many dimensions than in two-dimensional problems like these.

None of these models, including ours, capture certain forms of the compromise effect (Simonson 1989; Tversky and Simonson 1993). In our model, a person who is indifferent between 2-dimensional options c, c', and c'' without relative thinking will remain indifferent with relative thinking: he will not display a strict preference for the middle option. Likewise, Bordalo, Gennaioli, and Shleifer (2013) observe that their model does not mechanically generate a preference for choosing "middle" options. Kőszegi and Szeidl (2013) and Cunningham (2013) also do not generate these effects.[19]

An alternative interpretation for why trade-offs depend on ranges along consumption dimensions—giving rise to the sorts of choice-set effects we emphasize in this section—is that this follows as a consequence of inference from contextual cues, broadly in the spirit of mechanisms proposed by Wernerfelt (1995) and Kamenica (2008). In some circumstances, a person who is uncertain how to value an attribute dimension may rationally place less weight on that dimension when its range is wider, perhaps by guessing that hedonic ranges tend to be similar across dimensions, and therefore guessing that the hedonic interpretation for a change in a dimension is inversely related to the range in that unit. While we believe that such inference mechanisms likely play an important role in some situations, evidence suggests that they do not tell a very full story. There is evidence of range effects in trade-offs involving money and other dimensions that are easily evaluated, such as in Soltani, De Martino, and Camerer (2012), where people make choices between lotteries that vary in the probability and magnitude of gains. Mellers and Cooke (1994) show that range effects are found even when attributes have a natural range that is independent of the choice set, for

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[18]Soltani, De Martino, and Camerer (2012) present a model that shares similar motivations, but is written with a different focus: Their model shows how the biophysical limits of neural representations can account for range effects in a specific choice context. Our model instead takes range effects as given, but fleshes out the assumptions necessary to broaden the domain of application to a greater variety of economic contexts.

[19]For a review of models aimed at capturing the compromise effect, see Kivetz, Netzer, and Srinivasan (2004).

example when they represent percentage scores, which naturally vary between 0 and 100.

3.2 The Limits of Choice-Set Effects

We now provide bounds for the choice-set effects that result from relative thinking. Given any two options c and c', the following proposition supplies necessary and sufficient conditions on their relationship for there to exist a choice set under which c' is chosen over c.

Proposition 2.

1. *Assume that each $u_k(c_k)$ is unbounded above and below. For $c, c' \in \mathbb{R}^K$ with $U(c') \geq U(c)$, either c' would be chosen from $\{c, c'\}$ or there exists c'' that is undominated in $\{c, c', c''\}$ such that c' would be chosen from $\{c, c', c''\}$.*

2. *Assume that each $u_k(c_k)$ is unbounded below. For $c, c' \in \mathbb{R}^K$ with $U(c) \neq U(c')$ there is a C containing $\{c, c'\}$ such that c' is chosen from C if and only if*

$$\sum_{i \in A(c',c)} w(\delta_i(c',c)) \cdot \delta_i(c',c) + \sum_{i \in D(c',c)} w(\infty) \cdot \delta_i(c',c) > 0, \tag{1}$$

where $A(c',c) = \{k : u_k(c'_k) > u_k(c_k)\}$ denotes the set of c''s advantageous dimensions relative to c and $D(c',c) = \{k : u_k(c'_k) < u_k(c_k)\}$ denotes the set of c''s disadvantageous dimensions relative to c.

Part 1 of Proposition 2 shows that if c' yields a higher un-normed utility than c, then there exists some choice set where it is chosen over c. This part only relies on *N0(d)*, or in particular that the person makes a utility-maximizing choice from C whenever the range of utility on each dimension is the same given C, or whenever $\Delta_j(C)$ is constant in j.[20] The intuition is simple: so long as utility is unbounded, one can always add an option to equate the ranges across dimensions. For example, while we saw before that the relative thinker prefers $(2, 3, 0)$ over $(0, 0, 5)$ from a binary choice set, this finding says that because the un-normed utilities of the two options are equal there exists a choice set containing those options under which the relative thinker would instead choose $(0, 0, 5)$. In particular, a person would always choose $(0, 0, 5)$ from $\{(2, 3, 0), (0, 0, 5), (5, -2, 0)\}$.

The second part of the proposition uses the additional structure of Assumptions *N1-N2* to supply a necessary and sufficient condition for there to exist a comparison set containing $\{c, c'\}$ such that the person chooses c' over c.[21] Condition (1) is equivalent to asking whether c' would be chosen

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[20]As a result, the first part of Proposition 2 also holds for Kőszegi and Szeidl (2013).

[21]As the proof of Proposition 2 makes clear, the conclusions are unchanged if C is restricted such that each $c'' \in C \setminus \{c, c'\}$ is undominated in C.

over c when the comparison set is such that the range over its advantageous dimensions are the smallest possible (i.e., $u_i(c_i') - u_i(c_i)$), while the range over its disadvantageous dimensions are the largest possible (i.e., ∞). In the classical model (with a constant w_k), this condition reduces to $U(c') > U(c)$. In the limiting case—ruled out by *N3* – where $w(\infty) = 0$, the condition is that c' has *some* advantageous dimension relative to c (i.e., is not dominated). More generally, as spelled out in Appendix A.2, the difference in un-normed utilities between the options cannot favor c "too much" and c' must have some advantages relative to c that can be magnified.[22]

Taken together, the two parts of the proposition say that the impact of the comparison set is bounded in our model. In particular, if $U(c') > U(c)$, it is always possible to find a comparison set under which the agent displays a preference for c' over c. However, it is only possible to find a comparison set under which the agent displays a preference for c over c' when (1) holds.

Our model implies two additional limits to comparison-set effects. First, our model says that people maximize un-normed consumption utility when decisions involve sufficiently large stakes: Supposing each $u_k(\cdot)$ is unbounded, one can show that for all comparison sets C, there exists a $\bar{t} > 0$ such that if c' is an un-normed utility-maximizing choice from C, then $t \cdot c'$ is a normed utility-maximizing choice from $t \cdot C$ for all $t > \bar{t}$. One intuition is that absolute differences scale up with t, but proportional differences do not, so absolute differences dominate decision-making as t gets large.

Second, as shown in Appendix A.2 (Proposition 6), for any option c, there exists a choice set C containing c together with "prophylactic decoys" such that c will be chosen and, for any expansion of that set, only options that yield "approximately equivalent" un-normed utility to c or better can be chosen. This means, roughly, that once C is available no decoys can be used to leverage range effects to make a consumer choose any option inferior to c. With unbounded utility and Assumption *N3*, it is always possible to add options that make the ranges on dimensions sufficiently large such that further expanding the choice set will not make some dimensions receive much larger decision weights than others. One potential application of this result lies in examining competition in a product market. For example, a firm that wishes to sell some target product can always market other products that would not be chosen, but prevent other firms from introducing options that frame sufficiently inferior products as superior. This suggests that relative thinking may influence the options that are offered to market participants by more than it influences ultimate choices.[23]

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[22]These comparative statics can perhaps be seen more clearly by re-writing inequality (1) under *N3* as

$$U(c') - U(c) + \sum_{i \in A(c',c)} \left(\frac{w(\delta_i(c',c))}{w(\infty)} - 1 \right) \cdot \delta_i(c',c) > 0,$$

which highlights how the inequality depends on the difference in un-normed utilities as well as whether c' has advantages relative to c.

[23]These conclusions depend on the market structure as well as the technologies that are available to firms. Mon-

4 The Full Model

Section 2 presented the special case that our model reduces to when there is no uncertainty. We now present our full model that allows for uncertainty. Formally, the decision-maker chooses between lotteries on $\mathbb{R}^K$, with a choice set $\mathscr{F} \subset \Delta(\mathbb{R}^K)$. The deterministic case of Section 2 lends itself to a single notion of range (once a comparison set is specified), but elides some issues that arise in the case where the comparison set contains options that generate stochastic outcomes on one or more dimensions.

Suppose, for instance, that Nomi is a day laborer who must make a choice between working a known-to-be-available Job A that pays \$100 for 10 hours of work and a known-to-be-available Job B that pays \$110 for 12 hours of work. But she can also go stand in a queue for day laborers, where the job she gets when at the front of the queue is ex ante uncertain. The amount of hours is variable: with equal likelihood, the number of hours she will have to work is $11 \pm h$. The pay is variable: She will be paid \$55 with probability q, \$155 with probability r, and \$105 with probability $1 - q - r$, where (for the sake of simplicity) assume that the amount of money offered is independent from the number of hours offered. Nomi's intrinsic, un-normed utility is that each hour of work is worth \$10. If her only two options were *A* and *B*, our earlier results on deterministic choice say she would always choose *A* since this is a balanced choice: it is worth it to her to give up \$10 in income to save two hours of work. If she can also choose not to work, she will continue to choose *A*: she'd be indifferent between not working or choosing *A* from the binary choice set, but will strictly prefer choosing *A* when *B* is included since it expands the range on the disutility-of-working dimension by more than it expands the range on the utility-from-money dimension. But what will Nomi choose—Options A, B, Q, or not working—as a function of the value of h, q, and r? To answer such a question, we need to expand our definition of the range to handle stochastic outcomes.

The two simplest formulations, both of which would map onto the Section 2 definition in the deterministic case, are to look at the range of expected values of lotteries in the choice set in each dimension, or to take the highest and lowest possible outcomes that can occur in any lottery. The first can be formalized as taking the range equal to $\Delta_k^{\exp}(\mathscr{F}) = \max_{F \in \mathscr{F}} E_F[u_k(c_k)] - \min_{F \in \mathscr{F}} E_F[u_k(c_k)]$. The second can be formalized as taking the range equal to $\Delta_k^{\mathrm{supp}}(\mathscr{F}) = \max_{c \in \cup_{F \in \mathscr{F}} \mathrm{supp}(F)} u_k(c_k) - \min_{c \in \cup_{F \in \mathscr{F}} \mathrm{supp}(F)} u_k(c_k)$.

A problem in the first case (taking the range to equal $\Delta_k^{\exp}(\mathscr{F})$) is that only the range of outcomes across lotteries would matter, not the range of outcomes within lotteries. This formulation would

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polists may have an incentive to market decoys that get consumers to buy inferior products. In competitive situations, consumers may still buy inferior products if prophylactic decoys are prohibitively costly to market, for example if they must be built in order to market them or if there is sufficient consumer heterogeneity that some consumers will actually demand these products if they are offered. A more complete analysis of competition with decoys, which is left for future work, would need to grapple with such issues.

say, for example, that the amount of uncertainty Nomi faces in hours worked if she enters the queue (captured by the level of h) would not impact how she trades off hours worked against the level of pay. Intuitively, however, increasing this uncertainty would decrease Nomi's sensitivity to hours worked, for example making higher-paying and greater-hours-worked job B look more attractive relative to lower-paying and lower-hours-worked job A.

A problem in the second case (taking the range to equal $\Delta_k^{\text{supp}}(\mathscr{F})$) is that it would treat low and high probability outcomes the same. This formulation would say, for example, that the possibility that Nomi could get a job that pays \$155 by standing in the queue would impact her sensitivity to pay independently of the likelihood, r, of getting such a job. And the impact of the possibility of \$155 on the range would remain large as $r \to 0$. Intuitively, however, increasing this probability should decrease Nomi's sensitivity to pay, for example making higher-paying and greater-hours-worked job B look less attractive relative to lower-paying and lower-hours-worked job A. That is, a \$5 return to an hour of work looms small when a \$55 return to an hour of work is possible—and even smaller when a \$55 return to an hour of work is probable. And, while realistic non-linear probability weighting (Kahneman and Tversky 1979) could mitigate the following effect a bit, the impact of the possibility of the \$155 job on the range should vanish to zero as the probability of being offered such a job tends to zero.

Guided by such intuitions, our goal is to provide a formulation of ranges that satisfies the following criteria:

1. The range along a dimension depends on within-lottery ranges, not just between-lottery ranges.

2. The range along a dimension depends on probabilities, not just possible outcomes.

To satisfy these criteria, we take the following approach: We summarize every lottery's marginal distribution over $u_k(c_k)$ by the mean plus or minus its variation around the mean, where variation is measured by something akin to the standard deviation around the mean, and then take the range along a dimension to equal the difference between the maximal and minimal elements across the summarized distributions. The actual measure of variation we use is 1/2 the "average self-distance" of a lottery. We do not believe that a person literally calculates average self distance: Our use of this statistic is to approximate the idea that a person's perception of the range of outcomes is increasing in dispersion. We believe very little of our analysis would qualitatively change if classical standard deviation or other notions of dispersion were used.[24]

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[24]The average self-distance is a term Kőszegi and Rabin (2007) coined, but the identical measure had already been discussed in the literature under different terms, such as the *L*-scale or mean difference. Indeed, Yitzhaki (1982) argues that the mean difference provides a more central notion of statistical dispersion in some models of risk preference since it can be combined with the mean to construct necessary conditions for second-order stochastic dominance. Kőszegi

Formally, given a comparison set $\mathscr{F}$, we define the range along dimension k to equal

$$\Delta_k(\mathscr{F}) = \max_{F \in \mathscr{F}} \left(E_F[u_k(c_k)] + \frac{1}{2} S_F[u_k(c_k)] \right) - \min_{F \in \mathscr{F}} \left(E_F[u_k(c_k)] - \frac{1}{2} S_F[u_k(c_k)] \right), \qquad (2)$$

where $E_F[u_k(c_k)] = \int u_k(c_k) dF(c)$ equals the expectation of $u_k(c_k)$ under F, and $S_F[u_k(c_k)] = \int \int |u_k(c_k') - u_k(c_k)| dF(c') dF(c)$ is the "average self-distance" of $u_k(c_k)$ under F, or the average distance between two independent draws from the distribution. Note that the range along a dimension collapses to the previous riskless specification when all lotteries in $\mathscr{F}$ are degenerate. Note also that when $K = 1$ and $\mathscr{F} = \{F\}$ is a singleton choice set, then we have $\Delta(\mathscr{F}) = S_F[u(c)]$.[25]

We assume the weights w_k satisfy the following assumptions, which generalize the conditions from Section 2.

Norming Assumptions:

  N0. The weights w_k are given by $w_k = w(\Delta_k(\mathscr{F}))$, where $\Delta_k(\mathscr{F})$ is given by (2).

  N1. $w(\Delta)$ is a differentiable, decreasing function on $(0, \infty)$.

  N2. $w(\Delta) \cdot \Delta$ is defined on $[0, \infty)$ and is strictly increasing.

Assumption *N0* expands the definition of the range from *N0(d)* to the more general definition of (2), while the rest of the norming assumptions remain as they were in the deterministic case. Given the comparison set, the decision-maker evaluates probabability measure F over $\mathbb{R}^K$ according to:

$$U^N(F|\mathscr{F}) = \int U^N(c|\mathscr{F}) dF(c),$$

where $U^N(c|\mathscr{F}) = \sum_k w(\Delta_k(\mathscr{F})) \cdot u_k(c_k)$ as in the riskless case.

To build more intuition, we return to the Nomi example and additionally consider an example that connects to experimental paradigms on context-dependent preferences.

Example 1. Write Nomi's un-normed utility function as $U = m - 10 \cdot h$, where m is money and h is hours worked. Here, $c_1 = m$, $u_1(c_1) = c_1$, $c_2 = h$, $u_2(c_2) = -10 \cdot c_2$. Nomi chooses between four lotteries: not working (NW), taking job A, taking job B, and the lottery Q representing standing

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and Rabin (2007) show it to be especially relevant for models of expectations-based-reference-dependent utility; as such, using it as a measure of dispersion may facilitate combining our model with reference dependence.

[25] The proof of Lemma 1 in Appendix D establishes that, for any lottery F, $E[F] + 1/2 \cdot S(F) = E_F[\max\{c, c'\}]$, and we can similarly establish that $E[F] - 1/2 \cdot S(F) = E_F[\min\{c, c'\}]$. This provides an alternative expression for $\Delta_k(\mathscr{F})$:

$$\Delta_k(\mathscr{F}) = \max_{F \in \mathscr{F}} E_F[\max\{u_k(c_k), u_k(c_k')\}] - \min_{F \in \mathscr{F}} E_F[\min\{u_k(c_k), u_k(c_k')\}].$$

in the queue and taking whichever job is offered. To derive the range, we first write the expected value and average self-distance for each lottery:

$$(E_{NW}[u_1(c_1)], E_{NW}[u_2(c_2)]) = (0,0)$$
$$(S_{NW}[u_1(c_1)], S_{NW}[u_2(c_2)]) = (0,0)$$

$$(E_A[u_1(c_1)], E_A[u_2(c_2)]) = (100, -100)$$
$$(S_A[u_1(c_1)], S_A[u_2(c_2)]) = (0,0)$$

$$(E_B[u_1(c_1)], E_B[u_2(c_2)]) = (110, -120)$$
$$(S_B[u_1(c_1)], S_B[u_2(c_2)]) = (0,0)$$

$$(E_Q[u_1(c_1)], E_Q[u_2(c_2)]) = (105 + 50(r-q), -110)$$
$$(S_Q[u_1(c_1)], S_Q[u_2(c_2)]) = (100[q(1-q) + r(1-r)], 10h)$$

We now apply the formula, (2), for ranges:

$$\Delta_1(\mathscr{F}) = \max\{110, 105 + 50[r(2-r) - q^2]\}$$
$$\Delta_2(\mathscr{F}) = \max\{120, 110 + 5h\}.$$

There are several features of these ranges worth noting: (i) The range along the disutility-of-working dimension 2 is increasing in h: the range along a dimension depends on within-lottery ranges, not just between-lottery ranges. (ii) The range along the utility-from-money dimension 1 is increasing in r and decreasing in q: The range along a dimension depends on probabilities, not just possible outcomes. (iii) The range along the utility-from-money dimension 1 converges to 110 as $r \to 0$, $q \to 0$: Vanishingly low probability outcomes do not influence the range.

Example 2. The decision-maker makes plans knowing he faces choice set $\{c, c'\}$ with probability q and choice set $\{c, c''\}$ with probability $1 - q$, so $\mathscr{F} = \{(1,c), (q,c'; 1-q, c), (q,c; 1-q, c''), (q,c'; 1-q, c'')\}$. Supposing $c = (0,0)$, $c' = (-1,1)$, and $c'' = (-1,2)$, the range along the first dimension is $\Delta_1(\mathscr{F}) = 1$ and the range along the second is $\Delta_2(\mathscr{F}) = 2 - q^2 \in [1,2]$.

The first example derives the range of outcomes Nomi anticipates without fully solving what Nomi chooses. We leave that as a fun exercise for the reader. The second example shows that when the choice set people ultimately face is uncertain, they may be influenced by options that are not in the realized choice set. Return to Figure 2 and consider what happens when the comparison

set can differ from the realized choice set, which will allow us to analyze how the relative-thinker's choice between c and c' is influenced by having contemplated the possibility of being able to choose a *superior* option. Some experimentalists (e.g., Soltani, De Martino and Camerer 2012) induce such a wedge between choice and comparison sets by having subjects first evaluate a set of alternatives during an "evaluation period" and then quickly make a selection from a random subset of those alternatives during a "selection period". Consider the situation, for instance, where the person makes plans prior to knowing which precise choice set he will face and makes choices from $\{c, c'\}$ with probability $1 - q$ and makes choices from $\{c, c', c''\}$ with probability q—under this interpretation, Figure 2 illustrates the relative-thinker's choice when $\{c, c'\}$ is realized and $q > 0$. In the grey area to the right of the diagonal line (the blue area can be symmetrically analyzed), the addition of c'' to the comparison set expands the range of c''s advantageous dimension by more than it expands the range of its disadvantageous dimension, pushing the relative-thinker to choose c from $\{c, c'\}$.

These results connect with a smaller experimental literature that examines how making subjects aware of a third "decoy" alternative—that could have been part of the choice set but is not— influences their preferences between the two alternatives in their realized choice set. While the overall evidence seems mixed and debated, as far as we are aware the cleanest experiments from the perspective of our model—for example, that make an effort to control for rational inference from contextual cues—have found that adding "asymmetrically dominant" (or "close to dominant") decoys to the comparison set decreases experimental subjects' propensity to choose the asymmetrically dominated target when the decoy is not present in the choice set (e.g, Soltani, De Martino, and Camerer 2012). Relatedly, Jahedi (2011) finds that subjects are less likely to buy a good (for example, an apple pie) if they are aware that there was some probability they could have bought two of the same good for roughly the same price—for example, that there was some probability of getting a two-for-one deal on apple pies. Finally, these results are consistent with van den Assem, van Dolder, and Thaler's (2012) evidence that game show contestants are more willing to cooperate in a variant of the Prisoner's Dilemma when the gains from defecting are much smaller than they could have been: if any non-pecuniary benefits from cooperating are fairly flat in the stakes relative to the monetary benefits from defecting, making the stakes small relative to what they could have been also makes the benefits from defecting appear relatively small.[26]

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[26] A contrasting finding in the experimental literature has come to be labeled the "phantom decoy effect" (Pratkanis and Farquhar 1992): presenting a dominant option declared to be unavailable can bias choice towards the similar dominated option (Highhouse 1996; Pettibone and Wedell 2007; Tsetsos, Usher and Chater 2010). Some of this may be due to rational inference: taking an example inspired by Highhouse (1996), if job candidates vary in interview ability and test scores, then knowledge that a job candidate with extremely high interview ability and medium test scores is no longer on the market may provide a signal that interview ability is more important than test scores, leading a person to select a candidate with high interview ability and medium test scores over a candidate with medium interview ability and high test scores.

These results and examples also bring up a question we've been sidestepping: how sticky is the range? After all, a person may not follow through on a complete contingent plan formed with ranges $(\Delta_k)_{k=1}^{K}$ in mind if, at some contingency, those are no longer the ranges he has in mind. Return to the example of Nomi and suppose she is able to turn down a job after entering the queue, but has to make her decision quickly. Imagine that she could either be offered a job that pays \$155 with 11 hours of work or a job that pays \$105 with 11 hours of work ($q = 0, h = 0$). Suppose further that she is offered the job that pays \$105. In her quick decision of whether to accept the job is she still thinking about the possibility that she could have been offered \$155? We suspect she is, just like the experimental participants in the Soltani, De Martino, and Camerer (2012) study were clearly thinking the options that were potentially available in the evaluation period when they quickly made a choice between a subset of those options in the selection period. In such instances, the right way to apply our model is as in Example 2 above, where a person forms the range prior to knowing the exact choice set she faces.

But, as the case with models where lagged expectations influence choices (e.g., Koszegi and Rabin 2006), this leaves open the question of when precisely the range is formed. Suppose Nomi is waiting in a very long queue, and by waiting in the queue she gives up the possibility of choosing jobs A and B. In evaluating a job offer after waiting a long time, does she still think about features of jobs A and B (e.g., the possibility of working 12 hours in job B), or did the range of outcomes she considered adjust as she waited in line? Searching for answers to such qustions is an active area of research (see, e.g., Heffetz 2018). For now, analyst judgement is required in determining the moment that ranges sink in.

5 Tradeoffs Across Dimensions Under Uncertainty

This section draws out how uncertainty influences the rate at which people trade off utility across dimensions. Consider a simple example where somebody chooses how much effort to put into a money-earning activity, but also earns money from another stochastic choice. He has two consumption dimensions—money and effort—and his un-normed utility is given by

$$U = r \cdot e \pm k - f \cdot e,$$

where r equals the return to effort, $e \in \{0, 1\}$ denotes his level of effort, $\pm k$ indicates an independent 50/50 win-k/lose-k lottery, and f represents the cost to effort. A person maximizing expected utility would choose $e^* = 1$ if and only if $r/f \geq 1$. Notably, his effort is independent of k.

By contrast, our model says that increasing k will decrease effort: the more income varies, the smaller will seem an additional dollar of income from effort, and so the less worthwhile will be the

23

effort.

To see this, we utilize the formula for ranges in stochastic settings outlined in Section 4. The choice set is binary, and consists of the lotteries associated with effort $e = 0$ and $e = 1$. The maximum of the range along the money dimension is given by $E[r \cdot e \pm k] + 1/2 \cdot S[r \cdot e \pm k]$ for $e = 1$, which yields $r + 1/2 \cdot k$. The minimum of the range along the money dimension is given by $E[r \cdot e \pm k] - 1/2 \cdot S[r \cdot e \pm k]$ for $e = 0$, which yields $0 - 1/2 \cdot k$. So the range in consumption utility along the money dimension is $r + k$. Similar calculations give us that the range along the effort dimension is f.

Normed utility then equals

$$U^N = w(r+k) \cdot (r \cdot e \pm k) - w(f) \cdot f \cdot e,$$

so expected normed utility equals $EU^N = w(r+k) \cdot r \cdot e - w(f) \cdot f \cdot e$. The person then works so long as

$$\frac{r}{f} \geq \frac{w(f)}{w(r+k)},$$

where the right-hand side of this inequality is increasing in k, and greater than 1 for large enough k. This implies that the relative thinker is less likely to work when k is larger: increasing uncertainty on a dimension decreases his sensitivity to incremental changes in utility along that dimension.[27]

To state a more general result, for lotteries $H, H' \in \Delta(\mathbb{R})$, let $H + H'$ denote the distribution of the sum of independent draws from the distributions H and H'. Additionally, for lotteries $F_i \in \Delta(\mathbb{R})$, $i = 1, \ldots, K$, let $(F_1, \ldots, F_K) \in \Delta(\mathbb{R}^K)$ denote the lottery where each c_i is independently drawn from F_i.

Proposition 3. *Assume $K = 2$ and each $u_i(\cdot)$ is linear for $i = 1, 2$.*

1. *For $F_1, F_2 \in \Delta(\mathbb{R})$ and $G_1, G_2 \in \Delta(\mathbb{R}^+)$, if (F_1, F_2) is chosen from $\{(F_1, F_2), (F_1 - G_1, F_2 + G_2)\}$, then (F_1, F_2') is chosen from $\{(F_1, F_2'), (F_1 - G_1, F_2' + G_2')\}$ whenever F_2' is a mean-*

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[27]If marginal utility over money is convex, as is often assumed in explaining precautionary savings using the neo-classical framework, then income uncertainty will have the opposite effect. In such a case, expected marginal utility is increasing in income uncertainty, so higher income uncertainty will increase the propensity to exert effort to boost income. If the less conventional assumption of concave marginal utility is made, then more uncertainty would, as in our model, decrease the value on money. But in either case, the effects would be calibrationally small for modest increases in uncertainty. Loss aversion, by contrast, makes a bigger and less ambiguous opposite prediction to ours. Consider a situation in which people are able to commit in advance to take effort. Applying the concept of choice-acclimating equilibria from Kőszegi and Rabin (2007) and assuming linear consumption utility, loss aversion predicts no impact of higher income uncertainty on the propensity to exert effort: The decision is determined solely by consumption utility. Consider a second situation in which the opportunity to exert effort comes as a surprise and the person thus previously expected to not exert effort. In this case, loss aversion predicts that the presence of a 50/50 win-k/lose-k lottery over money increases the person's willingness to exert effort: Returns to effort are shifted from being assessed as increasing gains to partially being assessed as reducing losses.

preserving spread of F_2 and G_2' is a mean-preserving spread of G_2. Moreover, the choice is unique whenever $F_2' \neq F_2$ or $G_2' \neq G_2$.

2. *For $F_1, F_2 \in \Delta(\mathbb{R}^+)$, suppose the person faces the distribution over choice sets of the form $\{(0,0), (-\tilde{x}, \tilde{y})\}$ that is induced by drawing $\tilde{x}$ from F_1 and $\tilde{y}$ from F_2. If $(0,0)$ is preferred to realization $(-x, y)$ given the resulting comparison set, then $(0,0)$ is strictly preferred to $(-x, y)$ if instead the distribution over choice sets is induced by drawing $\tilde{x}$ from F_1 and $\tilde{y}$ from $F_2' \neq F_2$, where F_2' first order stochastically dominates a mean-preserving spread of F_2.*

Part 1 of Proposition 3 generalizes the above example, and says that if the person is unwilling to sacrifice a given amount from one dimension to the other, he will not do so if the second dimension is made riskier. Notably, the proposition extends the example by allowing the person to influence the amount of risk he takes. To illustrate, consider a simple modification of the example where exposure to risk goes up in effort, and utility equals $U = e \cdot (r \pm k) - f \cdot e$. The proposition says that, again, the worker is less likely to exert effort when the amount of income uncertainty, k, is higher: the wider range in the monetary dimension reduces the worker's sensitivity to incremental changes in money.

Part 2 says that a person becomes less willing to transfer a given amount of utility from one dimension to a second when the background distribution of potential benefits on the second dimension becomes more dispersed or shifted upwards. For example, suppose a person is indifferent between exerting effort e to gain \$100 if he made plans knowing \$100 is the return to effort. Then this person will not exert effort if, *ex ante*, he placed equal probability on earning \$50, \$100, or \$150. And he will be even less likely to exert effort for \$100 if he were *ex ante* almost sure to be paid \$150 for effort, since this further expands the range on the money dimension and makes earning \$100 feel even smaller.

Of course, Proposition 3 applies to tradeoffs beyond those involving effort and money. For example, let $U = c_1 + c_2$, where $c_1 \in \{0, 1\}$ represents whether the person has some good and $c_2 \in \mathbb{R}$ represents dollar wealth. In this case, Part 2 says that the person will be more likely to buy when prices are more uncertain *ex ante*. For example, if a person is indifferent between buying and not buying at price \$20 if he knew that \$20 would be the price, then he strictly prefers to buy at \$20 if, *ex ante*, he placed equal probability on the price being \$15, \$20, or \$25. Part 2 also says that the person will be more likely to buy at a fixed price when he expected higher prices. In other words, people's reservation prices will go up in expected prices.

Uncertainty not only makes the person less willing to transfer utility across dimensions, but also attenuates his response to incentives. To see this, we enrich the above example by allowing the person's effort choices to be continuous: The person chooses the amount of effort $e \in [0, 1]$ to engage in a project at cost $1/2 \cdot e^2$, where e equals the probability that a project will be successful.

If successful, the project yields return $r + y$, where $r > 0$ and y is a mean-zero random variable drawn according to distribution F with $y > -r$ for all y in the support of F. The person knows y prior to his choice of effort, but makes contingent plans only given knowledge of F.

The range on the money dimension is $r + 1/2 \cdot S(F)$ and the range on the effort dimension is $1/2$, so for each y the person chooses an amount of effort to solve

$$\max_{e \in [0,1]} w(r + 1/2 \cdot S(F)) \cdot e \cdot (r+y) - w(1/2) \cdot 1/2 \cdot e^2.$$

Taking first-order conditions, we solve for contingent effort choice

$$e(y) = \frac{w(r + 1/2 \cdot S(F))}{w(1/2)} \cdot (r+y). \tag{3}$$

As before, we see that the person works less hard when the *ex ante* range on returns is larger: $\partial e / \partial S(F) < 0$. The novel comparative static is that the person's effort choices are also less sensitive to the realized return when this range is larger: $\partial^2 e / \partial S(F) \partial (r+y) < 0$. For example, the effort choices of a person who places a 50/50 chance on getting paid an \$8 or \$10 piece-rate for a task will appear more sensitive to the realized piece rate than a person who initially places equal probability on \$7, \$8, \$9, \$10, and \$11.

Summarizing, in the context of effort decisions, the model makes several novel predictions in uncertain environments. First, a person is less likely to exert effort for a fixed return when *ex ante* or *ex post* income uncertainty is greater. Second, the same is true when the person expected to earn more when forming plans. Third, a person's effort choices are less sensitive to the realized return when he faced greater uncertainty in *ex ante* possible returns.

While we know of no direct field evidence speaking to our predictions, findings provide suggestive supporting evidence. For instance, insuring farmers against adverse weather shocks such as drought can increase their willingness to make high-marginal-return investments.[28] While this investment response could in principle result from risk aversion if investment returns negatively covary with the marginal utility of consumption—for example, investment in fertilizer could have a lower return when rainfall and consumption are low and the marginal utility of consumption is then high—it may also in part result from similar mechanisms to those we highlight: Our model predicts a positive investment response *even when investment returns are uncorrelated with the newly insured risk*, thus broadening the set of circumstances where we would expect expanding insurance coverage to boost profitable investment.

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[28]See, e.g., Karlan, Osei, Osei-Akoto, and Udry (2014), Cole, Gine, and Vickery (2017), and Mobarak and Rosenzweig (2012), who find that uncertainty reduction through insurance seems to increase investments such as fertilizer use and weeding.

Suggestive evidence also comes from laboratory findings on relative pay and labor supply.[29] Bracha, Gneezy, and Loewenstein (2015) examine how one's pay relative to pay they or others received impact decisions on whether and how hard to work. Most relevant to the current discussion, they find that the willingness to complete a task for a given wage is inversely related to previous wages offered for a related task. People are less likely to show up to complete a survey for either $5 *or* $15 if they were previously offered $15 to complete a related survey than if they were previously offered $5.[30] Although a cleaner test of our model's predictions would more directly manipulate expectations, this is consistent with the model if, as seems plausible, expectations of future wages are increasing in the size of previous wages: Increasing the probability attached to a higher wage offer increases the range attached to money, thereby increasing the reservation wage.

We provide more direct supportive evidence in the vignettes study of the next section.

6   Experiments

To explore some of the predictions of range-based relative thinking, we conducted a pair of experiments on Amazon's Mechanical Turk (henceforth MTurk). These experiments attempt to isolate the impact of ranges from other features, e.g., averages or medians, of choice contexts. That said, the studies are not meant to be definitive, but rather to help guide more complete and systematic investigations of range effects in economic decisions. The experiments also make concrete some of our more abstract theoretical predictions.

The basic premise underlying both experiments is depicted in the right panel of Figure 3. The left panel depects a classic decoy or attraction-effect design, where one of the dimensions is "good" and one "bad." This setup captures, for instance, choices that vary in quality and price, as well as the experimental setup we describe below. The two target options (in decoy-effect parlance) are depicted in green. As we described earlier in Section 3, one such attraction effect is that the addition of the blue decoy to the initial binary set leads participants to favor the quality-advantaged target. Another attraction effect is that the addition of the orange decoy to the initial binary set leads participants to favor the price-advantaged target.

We showed in Section 3 that range-based relative thinking predicts the attraction effects described above. The addition of the blue decoy expands the range along the price dimension to the

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[29]The implication of our model that uncontrollable noise to income reduces a person's incentive to exert effort to boost income is reminiscent of a basic assumption of the "expectancy theory" of motivation in psychology (Vroom 1964), which assumes that the lack of control over a performance outcome is demotivating. Tests of this assumption find mixed results. See, e.g., Sloof and van Praag (2008) for references and an experimental test finding that noise does not impact performance.

[30]And, as one would expect and as embedded in our model, people like high wages: People are more likely to show up to complete a survey for $15 if they were previously offered $15 than to show up to complete a survey for $5 if they were previously offered $5.

height of the blue, vertically-striped, rectangle and barely increases the range along the quality dimension, making participants less sensitive to differences in price relative to quality. Similarly, the addition of the orange decoy expands the range along the quality dimension to the width of the orange, horizontally-striped, rectangle and barely increases the range along the price dimension, making participants less sensitive to differences in quality relative to price. However, the addition of either the blue or orange decoy impacts not only the ranges along each dimension, but also other (perhaps important) features such as averages or medians. Indeed, it is not possible with the addition of a single decoy to vary the range without varying the average.[31]

Our experiments aim to more carefully isolate the impact of ranges. Figure 3b sketches the key idea. As in previous studies on attraction effects, we consider two target options designed such that participants are close to indifferent between them in a binary choice. However, since we seek to examine the impact of expanding ranges while fixing averages and medians, we do not explore the impact of adding a single decoy. Instead, we consider how the preference between the two target options differs in the context of the blue rectangle, where the range along the vertical dimension is wider than the range along the horizontal dimension, to the preference in the context of the orange rectangle where the range along the horizontal dimension is wider than the range along the vertical dimension. Our experiments provide several ways of instantiating this sort of design, making it clear how we trigger the blue and orange rectangles.[32]

6.1 Vignettes

First, we report results from a "vignettes" experiment in which participants answered a series of hypothetical-choice questions. These designs are motivated by Figure 3, but the real-effort experiment presented in the next section will be more directly linked to the figure. We recruited adults from the United States ($n = 500$; analysis sample $n = 492$) who had previously completed at least 100 prior tasks on MTurk with a 95% approval rating.[33] Participants were paid \$1.00 for

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[31]Thus, existing experiments in economics using such setups that provide evidence consistent (e.g., Soltani, de Martino, and Camerer 2012; Somerville 2019) and inconsistent with range-based relative thinking have not fixed other aspects of the choice context when varying ranges. Dertwinkel-Kalt et al. (2017) present experimental evidence from a design aimed at testing predictions of Bordalo, Gennaioli, and Shleifer (2012). Their results are supportive of Bordalo, Gennaioli, and Shleifer's predictions and inconsistent with predictions from the basic reference-independent version of our model. However, as the case with attraction-effect designs, in the Dertwinkel-Kalt et al. design salient aspects of the choice context are not fixed when varying ranges. This includes prices relative to expectations which are important not only in Bordalo, Gennaioli, and Shleifer (2012) but central to models such as Koszegi and Rabin (2006). This lack of control makes it difficult to draw strong conclusions about the existence or direction of range effects from their experiment.

[32]In the online appendix, we offer additional details and report all experiments conducted. We conducted two pilots aimed at calibrating magnitudes to ensure that a similar fraction of participants would choose each target option over the other when ranges were balanced across dimensions. In the online appendix, we also give instructions and materials for each experiment; the full data set is available by request.

[33]We remove from our analysis participants who failed an attention check.
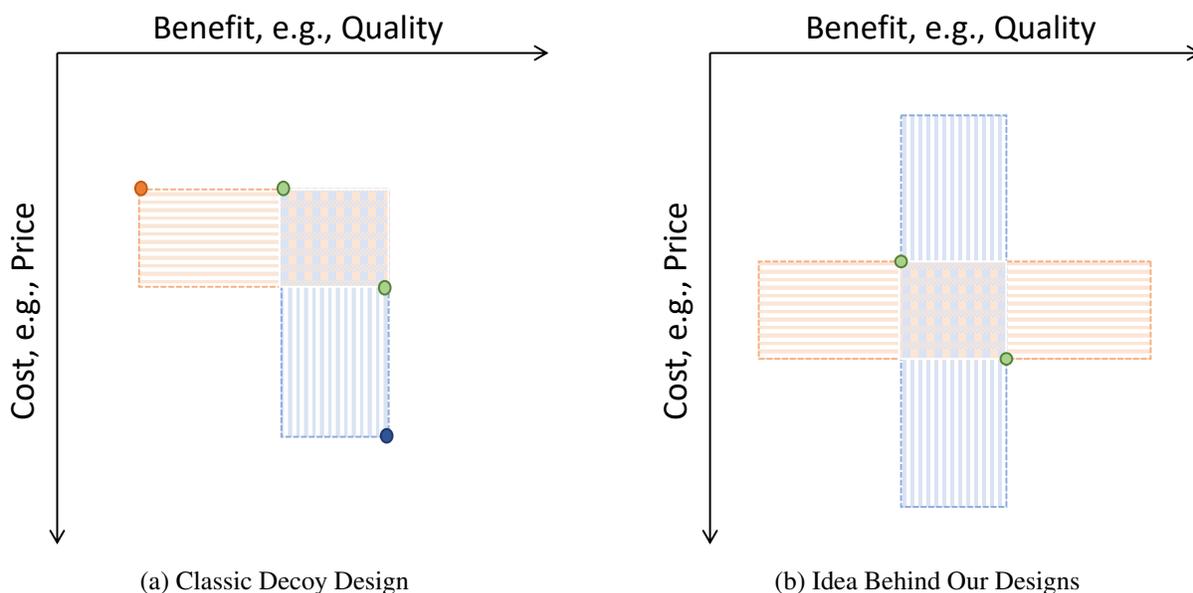
Figure 3: Isolating the impact of the range. The height and width of orange, horizontally-striped, rectangles depict narrow ranges on cost and wide ranges on quality that lead participants to view the cost-advantaged green target more attractive than the quality-advantaged green target. The height and width of blue, vertically-striped, rectangles depict wide ranges on cost and narrow ranges on quality that lead participants to view the quality-advantaged green target more attractive than the cost-advantaged green target.

answering three hypothetical questions of research interest as well as five demographic questions. The survey took an average of 11.7 minutes to complete (median completion time: 5.2 minutes). We asked participants each of three vignette-style questions (in randomized order).

The first question was a variant of Shah, Shafir, and Mullainathan's (2015) updated version of the famous jacket-calculator vignette (see Section 2). We included this question only to provide a base-line for behavior and to check whether the MTurk population's behavior matched measurements from other populations. Concerned with ceiling-effects cited in Shah, Shafir, and Mullainathan (2015), we reduced the size of the discount to $25, from $50 in their version. Our version of this question follows:

Imagine you have spent the day shopping. One item you have been shopping for is a laptop [*pair of headphones*]. At the end of the day, you find yourself at a store that has the brand and model you want for $1000 [*$100*]. This is a good price but not the best you have seen today. One store—a thirty minute detour on your way home—has it for $975 [*$75*]. Do you buy the $1000 laptop [*$100 headphones*] and go home, or do you instead decide to take the detour to buy it [*them*] for $975 [*$75*] at the other store?

29

We randomly assigned each participant to one of the two conditions described above.[34] When shopping for a laptop, 48% of participants ($n = 248$) said they would drive to the other store. When shopping for headphones, 73% of participants ($n = 244$) did so ($p < .001$ for difference in proportions).[35] We thus replicate existing findings for this paradigm.

Our second hypothetical question introduced a related thought experiment motivated by Figure 3b. This vignette induces wider or narrower ranges (analogous to the blue or orange rectangles in the figure) through manipulating *ex ante* uncertainty about available options.

> You went to a store to buy a phone, expecting to pay between $490 and $510 [*$190 and $810*] and expecting the trip to the store to last between 5 and 55 [*15 and 45*] minutes. You've spent 30 minutes at the store and it turns out the phone you want costs exactly $500. Right before buying it, the clerk at the check-out counter says you could save $5 on the phone by filling out a 15-minute survey, bringing the total time at the store up to 45 minutes and the total cost of the phone down to $495. Do you fill out the survey?

Each participant ex ante faced either a narrow range in money and a wide range in time or a wide range in money and a narrow range in time. We call these treatments "narrow money" and "wide money", respectively. Our model predicts that participants are more likely to say they would fill out the survey in the narrow-money treatment. Indeed, in the narrow-money condition (without brackets above; $n = 255$), 58% of participants were willing to complete the survey. In the wide-money condition ($n = 237$), 44% of participants were willing to complete the survey ($p = .0023$ for difference in proportions). This corresponds to an effect size of approximately 14 percentage points or roughly half the magnitude of the (well-studied and replicated) effect in our first vignette.

Our third vignette varied ranges with ex post uncertainty. We presented participants with a scenario in which another person had faced uncertainty over their wages and that uncertainty had not yet been resolved.

> Tarso has been working at the same restaurant for years and after a long shift, he is ready to go home. As he packs his things, he notices a group of four people enter and sit at a booth in the corner. He quickly thought to himself: should I stay or should I go? His work shares tips and he suspects that he would earn a $5 tip from the remaining table. Tarso is uncertain about his tips thus far, but suspects he earned between $35 and

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[34]As we detail in the online appendix, randomization produced balance on all characteristics we measured (age, income, and gender).

[35]All p-values are for two-sided tests for equality in proportions. Since our hypotheses are all directional, one-sided tests are arguably more appropriate. We report the p-values for two-sided tests to be conservative and follow convention.

$40 [*$15 and $60*] thus far from the pooled tips tonight. Therefore if he stays, Tarso thinks he'll earn between $40 and $45 [*$20 and $65*]. Do you think he: (a) stayed and served one final group; (b) left for the night?

Insofar as participants do a good job putting themselves in Tarso's shoes, our model says that participants will more frequently predict that Tarso will stay in the narrow-range (without brackets above) condition.[36] In this condition ($n = 236$), 67% of participants thought Tarso would stay and serve another table. In the wide-range condition ($n = 256$), 55% of participants thought Tarso would stay and serve another table ($p = .0054$ for difference in proportions). This corresponds to an effect size of approximately 12.2 percentage points.

Results from this experiment (summarized in Table 1) are in line with the predictions of our model of range-based relative thinking. The next experiment studies real-effort decisions and more directly instantiates the design sketched in Figure 3b.

Table 1: RESULTS, EXPERIMENT 1

	Drive for Discount		Survey After Purchase		Extra Work	
Variable	headphones	laptop	narrow $	wide $	narrow $	wide $
Choice = Yes	72.6%	47.5%	57.6%	43.9%	66.9%	54.7%
Observations	248	244	255	237	236	256

6.2   Real Effort

Our second design involved real-effort decisions. We recruited participants ($n = 570$) by the same criteria as in the vignettes experiment, basing the number of participants on a rough back-of-the-envelope power calculation. Participants were paid $1.00 for completing an initial block of work and were given the option to earn additional money based on their choices. The experiment took an average of 33 minutes to complete.

Each participant completed five initial tedious, but simple, real-effort tasks. The task consisted of counting characters in a matrix; see Figure A.1 in the Appendix for a visual depiction of the real-effort task. This initial learning session was designed to reduce any inference about the task difficulty from the set of questions we asked. For each task, a new matrix was presented to the

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[36]We present this vignette in third person because we guessed that many participants have limited experience waiting tables.

participant, and the symbol they were required to count changed.[37] On average, participants completed each initial task in about 55 seconds.

After completing the initial tasks, we then asked a single question about participants' willingness to complete more of the same task. Specifically, we asked participants their preferences over four combinations of additional tasks and additional money, trading off receiving more money for needing to do more tasks. Our experimental variation stemmed from varying the ranges of money and tasks that participants faced in a between-subjects manipulation.

In one treatment ("wide effort"), participants faced the following menu of options, which we label with letters here for ease of reference:

$$A : (\$2.80, 2 \text{ tasks})$$
$$B : (\$2.20, 14 \text{ tasks})$$
$$C : (\$2.80, 18 \text{ tasks})$$
$$D : (\$2.20, 30 \text{ tasks}).$$

In the second treatment ("wide money"), participants faced the following menu:

$$A' : (\$4.50, 14 \text{ tasks})$$
$$B : (\$2.20, 14 \text{ tasks})$$
$$C : (\$2.80, 18 \text{ tasks})$$
$$D' : (\$0.50, 18 \text{ tasks}).$$

In a given menu, the order of presentation of the options was randomized across participants. In "wide effort", effort ranges from 2 tasks to 30 tasks while money ranges between $2.20 and $2.80; in "wide money", money ranges from $0.50 to $4.50 while effort ranges from 14 tasks to 18 tasks. Critically, both treatments preserved the averages of the money and effort dimensions.

Figure 4 graphically depicts the menus, highlighting how the choice sets above instantiate the design sketched in Figure 3b. A notable feature of the menus is that there is one (obviously) best option in each (A and A') and one worst option (D and D'). If we simply elicited choices between the four alternatives, we would be unable to detect any effect of range. So instead we elicited *incentivized rankings* from participants. Specifically, participants ranked each alternative from 1 (best) to 4 (worst). Their ranking was implemented according to the following mechanism: we drew two of the four options at random, and the person was given their preferred option from those two. We explained this mechanism in simple language and presented a short quiz to ensure com-

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[37]We utilized this particular task to reduce the possibility of cheating and to make each individual task require effort, since the changing target requires significant focus.
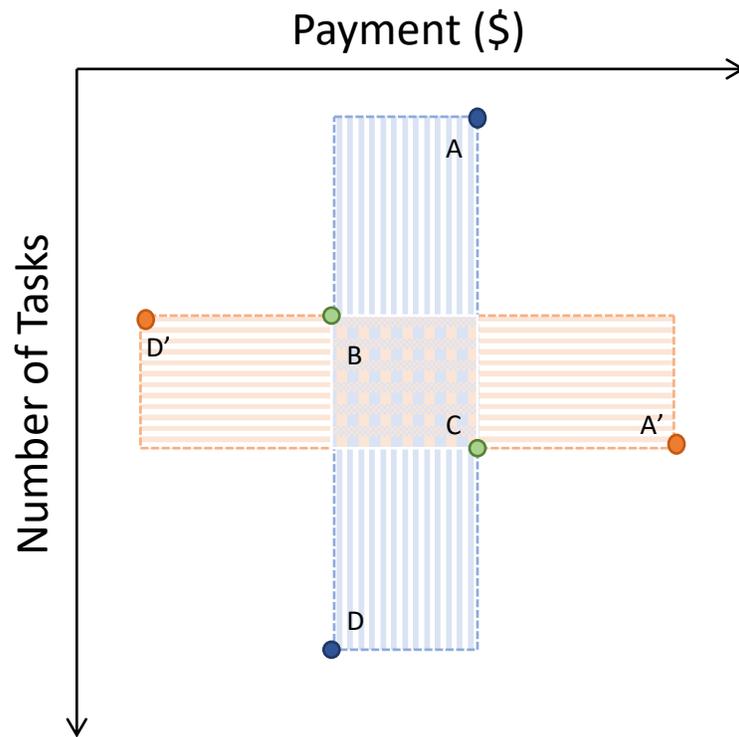
Figure 4: *Schematic of real-effort experiment design.* Participants ranked their preference from one of two sets: either $\{A,B,C,D\}$, which we call the *wide-effort* treatment, or $\{A',B,C,D'\}$, which we call the *wide-money* treatment.

prehension. There are two main ways participants may have interpreted the task: as heuristically providing a deterministic ranking or as literally choosing between lotteries over outcomes. Either way, our model of range-based relative thinking predicts that participants are more likely to rank C above B in the wide-effort treatment than in the wide-money treatment.

Before turning to results, we emphasize three key features of the design. First, as noted above, to isolate range-based relative thinking from average-based normalization (e.g., as emphasized in Cunningham 2013), the *average* attribute value along each dimension is fixed across treatments. Second, we utilize *single-question elicitation,* which limits the scope for theories of memory retrieval or cuing (e.g. Bordalo, Gennaioli and Shleifer 2019). Insofar as the base pay ($1) and number of initial tasks (5) serve as a norm for the wage range—perhaps suggesting that the task is "worth" $0.20—this norm remains constant across treatments. Third, decoy options feature *symmetric dominance*. The decoy options D, D' are dominated by *both* options and the decoy options A, A' dominate both options. This feature limits the scope for certain forms of reason-based choice that predict people will gravitate towards options that asymmetrically dominate an alternative or away from options that are asymmetrically dominated by an alternative.

Results from the experiment support the predictions of range-based relative thinking. The results are summarized in Table 2 and further analysis is provided in Online Appendix Table 2. We find that 46.3% of participants in the wide-effort treatment ranked C higher than B, while only 35.5% of participants in the wide-money treatment did so ($p = .0091$ for difference in proportions).

These results include responses from a small number of participants who ranked clearly dominated options first and dominant options last. We next consider results from a more restricted subgroup of participants that (1) rank the clearly best alternative (A or A') first; and (2) rank the clearly worst alternative (D or D') last. This restriction excludes 11% of the sample (65 participants). After conducting the experiment and analyzing both completed surveys and partial responses, we noticed that some participants seem to have attempted the survey multiple times as detected by IP addresses. For the sake of conservatism, exclude those participants as well, which removes an additional 34 participants.[38] We are left with a restricted sample of 471 participants. From this sample, approximately 48.3% of participants in the wide-effort condition ranked C as their second choice, while only 34.5% of participants in the wide-money condition did so ($p = .0023$ for difference in proportions).

Our experiments were designed to cleanly isolate range effects from other effects, but many alternative designs could help present a more complete picture of the economic importance of range-based relative thinking. Rather than attempting to isolate the impact of the range, such designs could instead focus on basic predictions of relative thinking. The model says, for example, that people behave "too similarly" across problems that are scaled up and down. Post, van den

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[38]Including these 34 participants does not directionally change our results.

Table 2: RESULTS, EXPERIMENT 2

Ranking	Wide Effort		Wide Money	
	Count	Percent	Count	Percent
B higher than C	158	54%	178	64%
C higher than B	136	46%	98	36%
$A \succ B \succ C \succ D$	132	45%	162	59%
$A \succ C \succ B \succ D$	126	43%	85	31%
B highest	12	4.1%	10	3.6%
C highest	4	1.4%	7	2.5%
Other	20	6.8%	12	4.3%
# of Participants (Whole Sample)	294		276	

Notes: Table 2 does not notationally distinguish between A and A' or D and D'. Percents do not sum to 100% due to rounding.

Assem, Baltussen, and Thaler (2008) provide suggestive evidence of such "scale insensitivity" in the context of choice under risk, which documented that risk aversion is more sensitive to stakes within than across contexts. Future work could explore scale sensitivity in other domains. For example, relative thinking predicts that an experimental participant's willingness to work for $5 vs. $10 would be less sensitive to the amount when evaluated separately rather than jointly. The intuition is that, separately, $5 and $10 would each represent 100% of the range on money, while, jointly, $5 would represent 50% of the range and $10 would represent 100% of the range. Is such a prediction borne out in the data?

7   Relationship to Focusing

The compelling intuition that a wider range of outcomes draws people's attention to a dimension (Bordalo, Gennaioli, and Shleifer 2012, 2013; Kőszegi and Szeidl 2013) is not the opposite of the idea that a wider range of outcomes decreases people's sensitivity to fixed changes within the range. To sharpen this intuition and to analyze the net effect of expanding the range, we sketch a framework for combining focusing and relative thinking effects.

As motivation, we believe some natural intuitions can guide speculation for when relative-thinking or focusing-effects may dominate. Our analysis above highlights examples where people's decisions are likely guided by a clear, low-dimensionality trade-off—between money and effort, money and quality, risk and return, etc. Per Figure 2 in Section 3 as well as the content of Section

6, we believe that many of the sharp, direct predictions of range-based focusing models contradict evidence and intuition in two dimensions. But both our analysis and the evidence we provide may "sample" only from situations where people's attention is directed to the relevant dimensions. In situations where dimensions may be neglected, the bigger-range-increases-incremental-weight hypothesis might be the dominant force, perhaps by approximating the idea that people stochastically notice or pay attention to dimensions according to their range.[39] In this light, it is notable that most of the examples provided by Kőszegi and Szeidl address trade-offs across many dimensions—and that virtually none of our examples consider more than 3 dimensions. With more dimensions, it becomes more intuitive that people may concentrate their attention on dimensions with wider ranges to the point of paying more attention to incremental changes along those dimensions.

This suggests that focusing effects might naturally arise in situations with many dimensions. A crude formulation might hold that people pay attention to the two dimensions with the greatest ranges, and make choices according to range-based relative thinking, applied as if those are the only two dimensions. Less crudely, we might suppose that other dimensions are partially attended to, but get decreased weight when their ranges are smaller. Consider the following formulation that channels this intuition, while maintaining the feature—both of our model and of Kőszegi and Szeidl (2013)—that people pay equal attention to dimensions when there are only two. First, order the dimensions $k = 1, 2, ...K$ (where K can be either finite or infinite) such that $\Delta_k \geq \Delta_{k+1}$. Then, choosing parameter $\chi \in [0,1]$, let ϕ_k be the *approximate focus weight* on dimension k as given by $\phi_{k+1} = \chi \cdot \phi_k$ for $k < K$, $\phi_K = \phi_{K-1}$, and $\sum_{k=1}^{K} \phi_k = 1$. The actual focus weights would be modified to take into account exact ties, where $\Delta_k = \Delta_{k+1}$. We denote the true focus weights by g_k such that $\sum_{k=1}^{j} g_k = \sum_{k=1}^{j} \phi_k$ for all j where $\Delta_j > \Delta_{j+1}$, and $g_j = g_{j+1}$ where $\Delta_j = \Delta_{j+1}$. Finally, we replace the weighting functions w_k, previously given by $w_k = w(\Delta_k(C))$. Instead, they are given by $w_k = g_k \cdot w(\Delta_k(C))$, where $w(\cdot)$ follows our Norming Assumptions *N0-N3*.

This implies (by brute-force construction) that range effects in two dimensional settings will be determined solely by relative thinking. But if there are at least three dimensions, the focus weights can matter. If we assume $\chi = .5$, for instance, three dimensions with utility ranges $(3, 2, 1)$ would get focus weights $(g_1, g_2, g_3) = (\frac{4}{8}, \frac{2}{8}, \frac{2}{8})$; dimensions with utility ranges $(3, 3, 1)$ would get focus weights $(g_1, g_2, g_3) = (\frac{3}{8}, \frac{3}{8}, \frac{2}{8})$; and dimensions with utility ranges $(3, 1, 1, 1, 1)$ would get focus weights $(g_1, g_2, g_3, g_4, g_5) = (\frac{4}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8})$. Comparing the first two examples illustrates that focus weights increase in the range when the increase influences the ranking of ranges across dimensions; comparing the second and third illustrates that this formulation shares Kőszegi and Szeidl's (2013) key feature that people pay less attention to advantages which are more spread out. In this (admittedly crude) formulation, relative thinking will dominate in two-dimensional choices

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[39]In models of inattention such as Gabaix (2014) and Schwartzstein (2014), the range of potential realizations on some dimension of concern might naturally be a determinant of the likelihood that people pay attention.

or whenever an increase in the range does not influence the ranking of ranges across dimensions. While big differences can draw attention (bigger Δ_k increases g_k), range-based relative thinking will dominate conditional on the allocation of attention (bigger Δ_k decreases w_k, conditional on g_k).

8   Discussion: Omissions and Shortcomings

A long line of psychological research treats range-based relative thinking as an important aspect of human perception and judgment, and we believe that the concomitant range-based choice behavior is relevant in economic decision-making. Yet the connection between the model we have developed and the broader economic implications is less clear, in part because the model is missing many other realistic factors in choice, and in part because it is incomplete in its rendering of the role of ranges in the psychology of choice.

The most conspicuous omission from our model is reference dependence, in the spirit of Kahneman and Tversky (1979). Indeed, other models of context effects embed aspects of this element. Variants of diminishing sensitivity have been incorporated into, e.g., Bordalo, Gennaioli, and Shleifer (2012) as well as Cunningham (2013), and the determination of a reference point is central to many context effects that do not rely on ranges.[40]

Incorporating reference dependence would enable us to study the *interaction* between relative thinking and both loss aversion as well as diminishing sensitivity. In ongoing work that first appeared in an early draft of this paper, we explore the former.[41] One intuition that emerges is that the availability of bigger and riskier choices may make a person act less risk averse. For example, if a homeowner is required to purchase some sort of insurance policy, adding policies with higher deductibles and lower premia to an existing menu could lead her to choose higher deductible policies. The analysis sheds light on evidence of certain context effects in risky choice, such as Post, van den Assem, Baltussen, and Thaler (2008), who show that many of the people playing the large-stakes (and skill-free) game show "Deal/No Deal" exhibit unusual insensitivity to risks over bets involving thousands of euros—risks that are small in the context of the game show.

Another intriguing way that relative thinking might interact with reference dependence is illus-

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[40]Another omitted feature of our framework (mentioned above when discussing the definitions of ranges) is non-linear probability weighting. People often seem to "overweight" both high and low probabilities in risky choice. This might be especially important in choice situations where a stochastic choice involves low probabilities: adding such low probabilities will expand the range slightly, and so in a sense be underweighted in our model.

[41]Quite how to integrate diminishing sensitivity into models of range effects is less clear. An intriguing possibility is that diminishing sensitivity itself is range-based. By that, we mean that it is possible that the wider the range in some dimension, the more muted the concavity over any particular absolute change in (un-normed) utility in that dimension. A variant of this idea could help accomodate natural forms of the compromise effect, in which people have a robust preference for middle options. Such compromise effects seem to be a range-based context effect—just one that is not captured by our particular range-based model.

trated by Walasek and Stewart (2015, 2018) who show how range effects influence the relative weights experimental participants place on losses versus gains.[42] In a range (as it were) of different hypothetical treatments, the authors gave people a large number of 50/50 gain/loss gambles in quick succession and asked them to press a button either accept (sometimes divided into weak and strong) or reject (also sometimes weak or strong). Across 10 treatments, gains and losses both ranged up to $20, $40, and $60 in various combinations—sometimes with higher ranges on gains, sometimes losses, and sometimes balanced. The authors were able to use ranges to change the coefficient of loss aversion—the ratio of value of a dollar decrease in loss vs. a dollar increase in gain—from 2.28 (for one of the narrow-loss-range conditions) to .81 (for one of the narrow-gain-range conditions).

Another class of elements missing from our model is framing effects, which is also missing in almost all other pinned-down formal models we are aware of. We've already implicitly illustrated examples of framing effects in some of the evidence we cite, where range effects are clearly in play but build from dimensions that are induced by particular experimental designs.[43] Formulating a model that does not allow for such framing effects has the disadvantage of missing large chunks of reality, but the advantage of highlighting forces that systematically hold across frames.

Another missing element in our analysis is not an omission that would alter its behavioral predictions, but rather its welfare interpretation. The interpretation that seems most consonant with our presentation—that relative thinking influences choice but not hedonic utility—seems realistic in many situations. However, in some situations it seems plausible that the way choices are hedonically experienced depends on how they were normed. For example, in situations involving risk, the difference between losing $10 and $5 may *feel* smaller when losing $100 was possible. The hedonic difference across different choice contexts—how does a person getting a given outcome feel when she had a wide range of possibilities vs. a narrow range—is even less easy to surmise. The model does not provide guidance on when relative thinking reflects a mistake or corresponds

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[42]They work with a framework called "Decision by Sampling", which can be thought of as a process model. The actual experimental manipulation was of ranges (and "frequencies"), which is how they framed their design and results. The titles of their papers are worth noting: "How to Make Loss Aversion Disappear and Reverse: Tests of the Decision [hyphen-*sic*] by [hyphen-*sic*] Sampling Origin of Loss Aversion" and "Context-Dependent Sensitivity to Losses: Range and Skew Manipulations" We report here only their (simpler) findings in the first of the papers, but the reader should note that the second paper shows (1) with skewness manipulations that the "frequency" also matters, and (2) can be interpreted as showing that the range effects were smaller (but still positive) than in the first paper, and (3) this time they could not make loss aversion go away.

[43]For example, Soltani, De Martino and Camerer (2012) appear to induce participants to treat (Probability Gain, Magnitude Gain) as dimensions by asking them to choose between lotteries of the form "gain *g* with probability *p*". Conversely, the studies by Walasek and Stewart (2015, 2018) appear to induce participants to treat gains and losses as separate dimensions by making the scales of losses and gains salient. Presumably different experiments and (more importantly) different real-world settings could induce different dimensions. For example, "states of the world" determining lottery payoffs could be made salient if all gambles are presented in terms of the dollar outcomes associated with heads and tails of the same coin.

to true experienced well-being.[44]

Our model shares some limitations with other theories of context-dependent preferences. The most conspicuous is the near-silence on the question of what exactly lies in the "comparison set". Rarely are the "comparison sets" posited in either our applications nor anywhere in the literature truly the choice sets that people face.[45] For example, a consumer can buy a car with or without a car radio—but she can also buy two cars at once. We think the intuition underlying most examples in papers on context-dependent preferences clearly relies on reasonable notions of what a person "might" do, and have tried to exclude any examples where our predictions are sensitive to options that might reasonably be added to the comparison set as far as we understand it—but it is not even clear without a conceptual framework quite what that means.[46]

A final limitation to the application of our model concerns its limited domain: We have articulated the model in static terms only. In ongoing work that appeared in a previous draft of this paper (Bushong, Rabin, and Schwartzstein 2017), we also extend the model to consider how a person trades off consumption across time. Applying the model to this environment requires assumptions about whether the person treats consumption at different points of time as different dimensions. We suppose that the person segregates out consumption today, but integrates consumption in future periods together into one dimension. Intuitively, the person thinks more precisely about how he spends money today than tomorrow.[47] Under these assumptions, the range of future consump-

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[44]We would conjecture that the psychology is probably in line with the most common interpretation of reference dependence and loss aversion: the experiential hedonics of deciding the outcome may be normed according to the ranges at the time of choice, but the utility experienced at the time of any later consumption is likely quite divorced from those ranges. This suggests that a hedonic interpretation of the norming may be most apt for smaller stakes (where later consumption utility is less relevant) than for larger ones (where later consumption utility is more relevant).

[45]Another limitation of the model seems more readily resolved by intuitions that most researchers would share: Models that are based on multiple dimensions must, obviously, have a notion of what those dimensions are. In understanding many of the motivating examples of both reference dependence and the sort of context effects we consider here, it is generally the case that experiential separability define the dimensions in a readily seen way. A mug is obviously different than money. People know how the quality of a good and its price are separate objects. A commonly shared mitigating intuition that is missing from our model (as well as most reference-dependent models) is the fact that there may be some psychological fungibility between dimensions: Big ranges on one dimension may make differences on *other* dimensions feel smaller.

[46]Treating the comparison set as exogeneous can also give an incomplete sense of how context-dependent preferences will impact choices in market situations, where firms have an incentive to influence the comparison set through the products that they market. In fact, we think the results on "prophylactic decoys" sketched above and formalized in Appendix A.2 should give pause on how researchers interpret evidence on decoys both in our brief discussion and in the extensive literature. In certain market contexts, these results indicate that, even though firms with inferior products might be able to use decoys to draw business away from a passive firm's superior product, the superior firm would prevail in an equilibrium where all firms can choose decoys. This suggests the importance of considering the market structure, as well as the marketing and production technologies, in thinking about how context-dependent preferences influence market outcomes.

[47]Rather than being a novel formulation, we view the assumption that people combine future spending into one aggregate "utility-of-spending dimension" as being implicit in other weighted-dimension models that treat "money" as a single dimension in static implementations. Indeed, like us, Kőszegi and Szeidl (2013) and Bordalo, Gennaioli, and Shleifer (2013) do not segregate money or price over multiple dimensions when they consider static versions

tion utility tends to be larger than the range of current consumption utility, so that relative thinking induces the person to act present-biased—the more so the longer the horizon of consumption and the greater the uncertainty in future consumption utility.

In some sense a more foundational question for fully specifying model dynamics concerns the speed with which ranges adjust and whether people correctly anticipate this speed. For example, in Section 4 we discussed people forming complete contingent plans of behavior, and commiting to only take certain actions. When they reach a given contingency, do they have the same ranges in mind that they did before? If not, did they appreciate this when they formed plans and made commitments? We suspect the answers to these questions are "not always" (e.g., with enough lag) and "no", respectively, but we leave the analysis for future research.

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of their models. This matters for some predictions. For example, Koszegi and Szeidl say that people necessarily maximize consumption utility when choosing whether or not to buy a single-dimensional good at a given price since this constitutes a "balanced choice" when money is treated as a single dimension. This result would not continue to hold under the alternative assumption that money could be treated as spread out over multiple temporal dimensions.

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Appendix Figure

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?	?	?	?	?	(	?	0	0	!!	i	0	?	?	!
!	?	!!	?	!!	!	?	?	?	!!	?	1	1	!	0
?	?	)	0	i	0	?	?	!!	?	!!	0	0	(	1
?	?	0	?	1	)	?	?	0	?	0	?	)	)	?
(	0	?	?	?	0	?	)	(	!!	?	(	0	?	0
)	i	0	i	0	0	0	i	i	(	0	?	0	0	0
(	(	t	1	t	(	0	!!	0	1	!	0	0	)	)

Symbol to count: ?

How many "?" are in the picture?

<<   >>

Figure A.1: *Screenshot of the counting task from the real-effort experiment.* The symbol to count as well as the matrix changed with each trial.

A Further Definitions and Results

A.1 Spreading Advantages and Disadvantages

Section 2 supplied examples on how the relative attractiveness of consumption vectors depends on the extent to which their advantages and disadvantages are spread out. To develop formal results, consider the following definition.

Definition 2. c'' *spreads out the advantages of* c' *relative to* c *if there exists a* $j \in E(c',c) = \{i : u_i(c_i') = u_i(c_i)\}$, $k \in A(c',c) = \{i : u_i(c_i') > u_i(c_i)\}$, *and* $\varepsilon < \delta_k(c',c)$ *such that*

$$(u_1(c_1''),\ldots,u_K(c_K'')) = (u_1(c_1'),\ldots,u_K(c_K')) + \varepsilon \cdot (e_j - e_k),$$

where e_i is the unit vector whose i'th element is 1. Analogously, c'' *integrates the disadvantages of* c' *relative to* c *if there exists* $j,k \in D(c',c) = \{i : u_i(c_i') < u_i(c_i)\}$ *such that*

$$(u_1(c_1''),\ldots,u_K(c_K'')) = (u_1(c_1'),\ldots,u_K(c_K')) + \delta_k(c',c) \cdot (e_j - e_k).$$

In words, c'' spreads the advantages of c' relative to c if c'' can be obtained from c' by keeping the total advantages and disadvantages relative to c constant, but spreading its advantages over a greater number of consumption dimensions. Conversely, c'' integrates the disadvantages of c' relative to c if c'' can be obtained from c' by keeping the total advantages and disadvantages relative to c constant, but integrating disadvantages spread over two dimensions into one of those dimensions.

Proposition 4. *If* c'' *spreads out the advantages of* c' *relative to* c *or integrates the losses of* c' *relative to* c, *then* $U^N(c'|\{c,c'\}) \geq U^N(c|\{c,c'\}) \Rightarrow U^N(c''|\{c,c''\}) > U^N(c|\{c,c''\})$.

Proposition 4 says that, all else equal, the attractiveness of one consumption vector over another goes up when its advantages are spread over more dimensions or its disadvantages are integrated. This connects to the evidence initially derived from diminishing sensitivity of the prospect theory value function that people prefer segregated gains and integrated losses (Thaler 1985), though the evidence on integrated losses (see Thaler 1999) is viewed as far less robust.[48] Thaler gives the

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[48]Note that, in contrast to diminishing sensitivity of the prospect theory value function, Proposition 4 does *not* imply the stronger result that the attractiveness of one consumption vector over another increases in the *degree* to which its advantages are spread or its losses are integrated. For example, while Proposition 4 implies that if $A = (x,0,0)$ is weakly preferred over $B = (0,0,y)$ from a binary choice set, then $A(\varepsilon) = (x-\varepsilon,\varepsilon,0)$ is strictly preferred over B from a binary choice set, it does not imply that if $A(\varepsilon)$ is weakly preferred over B, then $A(\varepsilon')$ is strictly preferred over B for $0 < \varepsilon < \varepsilon' < x/2$. The intuition is that, in moving from $(x,0,0)$ to $(x-\varepsilon,\varepsilon,0)$, A's advantage of x over B is unambiguously assessed with respect to a lower range: portion $x-\varepsilon$ of the advantage is assessed with respect to range $x-\varepsilon$ rather than x while portion ε is assessed with respect to range ε rather than x. On the other hand, in moving from $A(\varepsilon)$ to $A(\varepsilon')$, there is a trade-off where portion ε of the advantage is now assessed with respect to the *increased* range of ε'. Getting the unambiguous result appears to rely on further assumptions, for example that that $w(\Delta) \cdot \Delta$ is concave in Δ.

following example of a preference for segregated gains: when subjects are asked "Who is happier, someone who wins two lotteries that pay $50 and $25 respectively, or someone who wins a single lottery paying $75?" they tend to believe the person who wins twice is happer. This principle suggests, for example, why sellers of products with multiple dimensions attempt to highlight each dimension separately, e.g., by highlighting the many uses of a product in late-night television advertisements (Thaler 1985).

Turning to losses, Thaler (1985) asked subjects the following question:

> Mr. A received a letter from the IRS saying that he made a minor arithmetical mistake on his tax return and owed $100. He received a similar letter the same day from his state income tax authority saying he owed $50. There were no other repercussions from either mistake. Mr. B received a letter from the IRS saying that he made a minor arithmetical mistake on his tax return and owed $150. There were no other repercussions from his mistake. Who was more upset?

66% of subjects answered "Mr. A", indicating a preference for integrated losses. There is other evidence that urges some caution in how we interpret these results, however. Thaler and Johnson (1990) find that subjects believe Mr. A would be happier if the letters from the IRS and state income tax authority were received two weeks apart rather than on the same day. Under the assumption that events on the same day are easier to integrate, then this pattern goes against a preference for integrated losses. Similarly, while Thaler and Johnson find that subjects say a $9 loss hurts less when added to a $250 loss than alone (consistent with a preference for integrating losses), they also say that it hurts *more* when added to a $30 loss than alone (inconsistent with such a preference). While the overall evidence appears broadly consistent with the predictions of Proposition 4, the evidence on losses is ambiguous.

The model more broadly implies that it is easier to advantageously frame items whose advantages are more spread out:

Proposition 5. *Assume that $u_k(\cdot)$ is unbounded below for each k. Let $c, c', c'' \in \mathbb{R}^K$ where c'' spreads out the advantages of c' relative to c. Supposing there is a C containing $\{c, c'\}$ such that c' is chosen from C, then there is a $\tilde{C}$ containing $\{c, c''\}$ such that c'' is chosen from $\tilde{C}$.*

A.2 Further Results on the Limits of Choice-Set Effects and Prophylactic Decoys

Examining the necessary and sufficient condition (1) yields the following corollary:

Corollary 1.

1. *If c dominates c', where $c, c' \in \mathbb{R}^K$, then there does not exist a C containing $\{c, c'\}$ such that c' would be chosen from C.*

2. *Consider $c, c' \in \mathbb{R}^K$ where the total advantages of c' relative to c satisfy $\delta_A(c', c) \equiv \sum_{i \in A(c', c)} \delta_i(c', c) = \tilde{\delta}_A$ for some $\tilde{\delta}_A > 0$. Then, additionally assuming N3, there exists a finite constant $\bar{\delta} > 0$ for which there is a C containing $\{c, c'\}$ such that c' is chosen from C only if the total disadvantages of c' relative to c satisfy $\delta_D(c', c) \equiv -\sum_{i \in D(c', c)} \delta_i(c', c) < \bar{\delta}$.*

The first part of the corollary says that dominated options can never be framed in a way where they will be chosen over dominating alternatives. The second says that it is only possible to frame an inferior option in a way that it is chosen over a superior alternative if its disadvantages are not too large relative to its advantages.

The previous result establishes one way in which the impact of the comparison set is bounded in our model. The next result establishes another: for any option c, there exists a choice set containing c such that c will be chosen and, for any expansion of that set, only options that yield "roughly equivalent" utility to c or better can be chosen. Recalling that $\delta_A(c', c) = \sum_{i \in A(c', c)} \delta_i(c', c)$ and $\delta_D(c', c) = -\sum_{i \in D(c', c)} \delta_i(c', c)$, we have the following result:

Proposition 6. *Assume N3 and that $u_k(\cdot)$ is unbounded below for each k. For any $c \in \mathbb{R}^K$ and $\varepsilon > 0$, there exists some C_ε containing c such that the person would be willing to choose c from C_ε and would not choose any $c' \in \mathbb{R}^K$ with $\delta_A(c', c) = 0$ or $\delta_A(c', c) > 0$ and*

$$\frac{\delta_D(c', c)}{\delta_A(c', c)} - 1 > \varepsilon$$

from any $\tilde{C}$ containing C_ε.

Proposition 6 says that, for any option c, it is possible to construct a choice set containing c as well as "prophylactic decoys" that would not be chosen, but prevent expanding the choice set in ways that allow sufficiently inferior options to c to be framed as being better. With unbounded utility, it is always possible to add options that make the ranges on dimensions sufficiently large such that further expanding the choice set will not make some dimensions receive much larger decision weights than others. For example, if $c = (1, 8, 2)$ and $\bar{u} > 0$, then c is chosen from $C = \{(1, 8, 2), (1, 8 - \bar{u}, 2 - \bar{u}), (1 - \bar{u}, 8, 2 - \bar{u}), (1 - \bar{u}, 8 - \bar{u}, 2)\}$ and, as $\bar{u} \to \infty$, it is impossible to expand C in a way that significantly alters the ranges along various dimensions and allows an inferior option to c to be chosen.

These ideas may be seen more clearly when we start from two options rather than one. A simple corollary is that when one option c has a higher un-normed utility than another c', it is possible to find a comparison set including those options such that the person chooses c from that set and where it is not possible to expand the set in a way that will reverse his preference.

49

Corollary 2. *Assume the conditions of Proposition 6 hold. For any $c, c' \in \mathbb{R}^K$ with $U(c) > U(c')$, there exists some C containing $\{c, c'\}$ such that the person would be willing to choose c from C and would not choose c' from any $\tilde{C}$ containing C.*

Again, applying the result to think about product market competition, this result says that if a firm has a superior product to a competitor then, with unbounded utility, it can always add inferior decoys that lead the consumer to choose its target product, and prevent the competitor from adding decoys that frame its inferior product as superior.[49] To take an example, consider $c = (8, 2)$ and $c' = (4, 7)$. For concreteness, we could imagine cars where c has better speed and c' has better comfort. Starting from a binary choice set, the speedy car producer may be able to get consumers to buy its inferior product by adding similarly speedy but really uncomfortable decoy cars. However, Corollary 2 tells us that the comfortable car producer can always add prophylactic decoy cars that prevent the speedy car producer from being able to do this. These prophylactic decoys, such as $(-\bar{u}, 7.1)$ for $\bar{u}$ large, would "double-down" on the comfortable car's speed disadvantage, protecting this disadvantage from being framed as all that bad.[50]

B Eliciting Model Ingredients from Behavior

This section outlines an algorithm for eliciting $u_k(\cdot)$ and $w(\cdot)$ from behavior. The elicitation essentially follows the steps laid out by Kőszegi and Szeidl (2013) to elicit the ingredients of their model; we will closely follow their presentation. Their algorithm works for us because our model shares the feature that people make consumption-utility-maximizing choices in "balanced" decisions, which allows us to elicit consumption utility by examining choices in such decisions. We then elicit the weighting function $w(\cdot)$ by examining how bigger ranges influence the person's sensitivity to given differences in consumption utility.

We assume *N0(d)* and *N2* (but do not impose *N1*) and follow Kőszegi and Szeidl (2013) by assuming that we know how options map into attributes, that we can separately manipulate individual attributes of a person's options, and that the utility functions $u_k(\cdot)$ are differentiable. We also, without loss of generality, normalize $u_k(0) = 0$ for all k, $u'_1(0) = 1$, and $w(1) = 1$. We depart from Kőszegi and Szeidl (2013) by assuming $w(\Delta) \cdot \Delta$ is strictly increasing (Assumption *N2*), while they make the stronger assumption that $w(\Delta)$—or $g(\Delta)$ in their notation—is strictly increasing. We

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[49]The key assumption is that the superior firm can add decoys that make the range on its disadvantageous dimensions sufficiently large that the inferior firm cannot add its own decoys that significantly magnify the relative weight placed on its advantageous dimensions. This can be satisfied with bounded utility as well, so long as lower bounds of utilities along the superior firm's advantageous dimensions weakly exceed lower bounds along its disadvantageous dimensions. If we were to relax assumption *N3* that $w(\infty) > 0$ then, with unbounded utility, we could instead observe a form of "instability" where it is possible to expand any set C from which c is chosen so that c' is chosen and vice-versa.

[50]We suspect a similar result also holds for Kőszegi and Szeidl's (2013) model under natural restrictions on the "focusing weights", though the prophylactic decoys would look different.

will see that their elicitation algorithm still works under our weaker assumption and, in fact, their elicitation can be used to test our assumption that $w(\Delta)$ is decreasing against theirs that $w(\Delta)$ is increasing.

The first step of the algorithm is to elicit the utility functions $u_k(\cdot)$. Restricting attention to dimensions 1 and k, consider choice sets of the form

$$\{(0, x+q), (p, x)\}$$

for any $x \in \mathbb{R}$ and $p > 0$. For $p > 0$, set $q = q_x(p)$ to equal the amount that makes a person indifferent between the two options, so

$$w(u_1(p) - u_1(0)) \cdot (u_1(p) - u_1(0)) = w(u_k(x+q_x(p)) - u_k(x)) \cdot (u_k(x+q_x(p)) - u_k(x)),$$

which because $w(\Delta) \cdot \Delta$ is strictly increasing in Δ, implies that

$$u_1(p) - u_1(0) = u_k(x+q_x(p)) - u_k(x).$$

Dividing by p and letting $p \to 0$ yields

$$u_1'(0) = u_k'(x) \cdot q_x'(0),$$

which (using the normalization that $u_1'(0) = 1$) gives $u_k'(x)$, and (using the normalization that $u_k(0) = 0$) gives the entire utility function $u_k(\cdot)$. Intuitively, this step of the algorithm gives us, for every x, the marginal rate of substitution of attribute 1 for attribute k at $(0,x)$—this is $q_x'(0) = u_1'(0)/u_k'(x)$—which yields the entire shape of $u_k(x)$ given the normalization that $u_1'(0) = 1$. We can then similarly recover $u_1(\cdot)$ through using the elicited utility function for some $k > 1$.

The second step of the algorithm elicits the weights $w(\cdot)$, where we can now work directly with utilities since they have been elicited. Focus on dimensions $1, 2,$ and 3, and consider choice sets of the form

$$\{(0, 0, x_0), (1, x-p, 0), (1-q, x, 0)\},$$

for any $x \in \mathbb{R}^+$, where $x_0 > 0$ is sufficiently low that $(0,0,x_0)$ will not be chosen and whose purpose is to keep this option from being dominated by the others and from lying outside the comparison set (this is the only step of the algorithm where having more than two attributes matters). For some $p \in (0, x)$, we now find the $q = q_x(p)$ that makes the person indifferent between the second two

51

options in the choice set, requiring that p is sufficiently small that $q_x(p) < 1$, so

$$w(1) \cdot 1 + w(x) \cdot (x - p) = w(1) \cdot (1 - q_x(p)) + w(x) \cdot x.$$

This implies that $w(x) \cdot p = w(1) \cdot q_x(p)$ and, by the normalization $w(1) = 1$, gives us

$$w(x) = \frac{q_x(p)}{p}.$$

In this manner, we can elicit the entire weighting function $w(\cdot)$. Intuitively, for all x, this step of the algorithm elicits the marginal rate of substitution of utils along a dimension with weight $w(x)$ for utils along a dimension with weight $w(1)$, which yields exactly $w(x)$ given the normalization $w(1) = 1$.

With this elicited weighting function, we can, for example, test our assumption that $w(\cdot)$ is decreasing against Kőszegi and Szeidl's (2013) that $w(\cdot)$ is increasing. To illustrate, suppose dimensions 1 and 2 represent utility as a function of the number of apples and oranges, respectively, where utility is elicited through the first step of the algorithm. Ignoring the third dimension for simplicity, if we see that the person strictly prefers $(1/2 \text{ utils apples}, 3 \text{ utils oranges})$ from the choice set

$$\{(0 \text{ utils apples}, 0 \text{ utils oranges}), (1 \text{ utils apples}, 2.5 \text{ utils oranges}), (1/2 \text{ utils apples}, 3 \text{ utils oranges})\},$$

then $w(3)/w(1) > 1$, consistent with Kőszegi and Szeidl (2013), while if the person instead strictly prefers $(1 \text{ utils apples}, 2.5 \text{ utils oranges})$ from this choice set, then instead $w(3)/w(1) < 1$, consistent with our model.

C More Detailed Comparison to Other Models

As noted in the introduction, the basic feature of our model —that a given absolute difference looms smaller in the context of bigger ranges (Parducci 1965)—is not shared by Bordalo, Gennaioli and Shleifer (2012, 2013) or other recent approaches by Kőszegi and Szeidl (2013) and Cunningham (2013), who model how different features of the choice context influence how attributes of different options are weighed. To enable a detailed comparison between the approaches, we present versions of their models using similar notation to ours, and compare the models in the context of simple examples along the lines of the one introduced in Section 3.

All of these models share the feature that there is some $U(c) = \sum_k u_k(c_k)$ that is a person's consumption utility for a K-dimensional consumption bundle c, while there is some $\hat{U}(c|C) =$

$\sum_k w_k \cdot u_k(c_k)$ that is the "decision consumption utility" that he acts on. The models by Bordalo, Gennaioli and Shleifer (2012, 2013), Kőszegi and Szeidl (2013), and Cunningham (2013) differ from each other's, and from ours, in how they endogeneize the "decision weights" $w_k > 0$ as functions of various features of the choice context, and possibly the option c under consideration. Specifically, their models assume the following:

Alternative Model 1 (Bordalo, Gennaioli and Shleifer (2012, 2013).)**.** Bordalo, Gennaioli, and Shleifer's (2013) model of salience in consumer choice says that for option c, $w_i > w_j$ if and only if attribute i is "more salient" than j for option c given "evoked" set C of size N, where "more salient" is defined in the following way. Ignoring ties and, for notational simplicity, assuming positive attributes (each $u_k(c_k) > 0$), attribute i is more salient than j for c if $\sigma_i(c|C) > \sigma_j(c|C)$, where $\sigma_k(c|C) \equiv \sigma\left(u_k(c_k), \frac{1}{N}\sum_{c' \in C} u_k(c'_k)\right)$ and $\sigma(\cdot, \cdot)$, the "salience function", is symmetric, continuous, and satisfies the following conditions (thinking of $\bar{u}_k = \bar{u}_k(C) \equiv \frac{1}{N}\sum_{c' \in C} u_k(c'_k)$):

1. *Ordering.* Let $\mu = \text{sign}(u_k - \bar{u}_k)$. Then for any $\varepsilon, \varepsilon' \geq 0$ with $\varepsilon + \varepsilon' > 0$,

$$\sigma(u_k + \mu\varepsilon, \bar{u}_k - \mu\varepsilon') > \sigma(u_k, \bar{u}_k).$$

2. *Diminishing Sensitivity.* For any $u_k, \bar{u}_k \geq 0$ and for all $\varepsilon > 0$,

$$\sigma(u_k + \varepsilon, \bar{u}_k + \varepsilon) \leq \sigma(u_k, \bar{u}_k).$$

3. *Homogeneity of Degree Zero.* For all $\alpha > 0$,

$$\sigma(\alpha \cdot u_k, \alpha \cdot \bar{u}_k) = \sigma(u_k, \bar{u}_k).$$

The ordering property implies that, fixing the average level of an attribute, salience is increasing in absolute distance from the average. The diminishing-sensitivity property implies that, fixing the absolute distance from the average, salience is decreasing in the level of the average. Note that these two properties can point in opposite directions: increasing $u_i(c)$ for the option c with the highest value of $u_i(\cdot)$ increases $(u_i(c) - \bar{u}_i)$, suggesting higher salience by ordering, but also increases $\bar{u}_i$, which suggests lower salience by diminishing sensitivity. Homogeneity of degree zero places some structure on the trade-off between these two properties by, in this example, implying that ordering dominates diminishing sensitivity if and only if $u_i(c)/\bar{u}_i$ increases.

More generally, using Assumptions 1-3, it is straightforward to show the following:[51]

$$\text{For option } c, \ w_i > w_j \Leftrightarrow \sigma_i(c|C) > \sigma_j(c|C) \Leftrightarrow \frac{\max\{u_i(c_i), \bar{u}_i(C)\}}{\min\{u_i(c_i), \bar{u}_i(C)\}} > \frac{\max\{u_j(c_j), \bar{u}_j(C)\}}{\min\{u_j(c_j), \bar{u}_j(C)\}},$$

<div align="right">(BGS)</div>

where the level of w_i depends only on the salience rank of attribute i for option c in comparison set C.[52] An interpretation of condition (BGS) is that attribute i of option c attracts more attention than attribute j and receives greater "decision weight" when it "stands out" more relative to the average level of the attribute, where it stands out more when it is further from the average level of the attribute in proportional terms.

Alternative Model 2 (Kőszegi and Szeidl (2013)). Kőszegi and Szeidl's (2013) model of focusing specifies that the decision weight on attribute k equals

$$w_k = g(\Delta_k(C)), \ g'(\cdot) > 0, \tag{KS}$$

where $\Delta_k(C) = \max_{c' \in C} u_k(c'_k) - \min_{c' \in C} u_k(c'_k)$ equals the range of consumption utility along dimension k, exactly as in our model. However, the weight on a dimension is assumed to be *increasing* in this range, $g'(\Delta) > 0$, which directly opposes Assumption *N1* of our model. An interpretation of condition (KS) is that people focus more on attributes in which options generate a "greater range" of consumption utility, leading people to attend more to fixed differences in the context of bigger ranges.

Alternative Model 3 (Cunningham (2013)). Cunningham (2013) presents a model of relative thinking in which a person is less sensitive to changes on an attribute dimension when he has encountered larger absolute values along that dimension. Cunningham's model is one in which previous choice sets, in addition to the current choice set, affect a person's decision preferences, so we need to make some assumptions to compare the predictions of his model to ours, and in particular to apply his model when a person's choice history is unknown. We will apply his model assuming that the person's choice history equals his current choice or comparison set C. It is then

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[51] As Bordalo, Gennaioli and Shleifer (2013) discuss, Assumption 2 (Diminishing Sensitivity) is actually redundant given Assumptions 1 and 3 (Ordering and Homogeneity of Degree Zero).

[52] Specifically, letting $r_i(c|C) \in \{1, \ldots, K\}$ represent the salience rank of attribute i for option c given comparison set C (the most salient attribute has rank 1), Bordalo, Gennaioli and Shleifer (2013, Appendix B) assume that the weight attached to attribute i for option c is given by

$$w_i = \frac{\delta^{r_i(c|C)}}{\sum_k \delta^{r_k(c|C)}},$$

where $\delta \in (0, 1]$ inversely parameterizes the degree to which the salience ranking matters for choices.

in the spirit of his assumptions that the decision weight attached to attribute k equals[53]

$$w_k = f_k(|\bar{u}_k(C)|),\ f_k'(\cdot) < 0\ \forall k, \tag{TC}$$

where $\bar{u}_k(C) = \frac{1}{N}\sum_{c' \in C} u_k(c_k')$ is the average value of attribute k across elements of C and N is the number of elements in C. Formulation (TC) says that a person is less sensitive to differences on an attribute dimension in the context of choice sets containing options that, on average, have larger absolute values along that dimension.

To illustrate differences between the models, return to the example introduced in Section 3. Suppose a person is deciding between the following jobs:

Job X. Salary: 100K, Days Off: 199

Job Y. Salary: 110K, Days Off: 189

Job Z. Salary: 120K, Days Off: 119,

where his underlying utility is represented by $U = \text{Salary} + 1000 \times \text{Days Off}$. First, we will consider the person's choice of jobs when he is just choosing between X and Y, and then we will consider his choice when he can also choose Z.

As noted in Section 3, our model predicts that the person will be indifferent between jobs X and Y when choosing from $\{X,Y\}$, but instead strictly prefers the higher salary job Y when choosing from $\{X,Y,Z\}$. None of the three other models share our prediction in this example. The predictions of Kőszegi and Szeidl's (2013) model were presented in Section 3. Bordalo, Gennaioli, and Shleifer's (2013) predicts that a person will strictly prefer choosing the higher salary job Y from $\{X,Y\}$: Using condition (BGS), we see that salary is more salient than days off for both options in $\{X,Y\}$— salary is more salient than days off for X since $105/100 > 199/194$, and salary is more salient than days off for Y since $110/105 > 194/189$—so the person places greater decision weight on salary and chooses the higher salary option. Intuitively, by diminishing sensitivity, a 5K utility difference relative to the average on the salary dimension stands out more than a 5K utility difference relative to the average on the days off dimension, as the average on the salary dimension is lower. Like Kőszegi and Szeidl (2013), Bordalo, Gennaioli, and Shleifer (2013) predict that the person will reverse her choice to X from $\{X,Y,Z\}$: Using condition (BGS), the addition of Z leads days off to be salient for all options—days off is more salient than salary for X since $199/169 > 110/100$,

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[53]Cunningham (2013) considers a more general framework where utility is not necessarily separable across dimensions, and makes assumptions directly on marginal rates of substitution. Part of his paper considers implications of weaker assumptions on how "translations" of histories along dimensions influence marginal rates of substitution, rather than average levels of attributes along dimensions. We focus on his average formulation because it enables sharper predictions across a wider range of situations: it is always possible to rank averages, but not always possible to rank histories by translation.

days off is more salient than salary for Y since $189/169 > 110/110$, and days off is more salient than salary for Z since $169/119 > 120/110$—so the person places greater decision weight on days off and chooses X. Intuitively, their model says that the addition of job Z, which is a relative outlier in terms of days off, causes the days off of the various options to really stand out. Like Kőszegi and Szeidl (2013), the salience-based prediction of Bordalo, Gennaioli, and Shleifer (2013) in this two-dimensional example seems at odds with intuition generated from laboratory evidence on attraction or range effects.[54]

Cunningham's (2013) formulation does not pin down what a person chooses from $\{X,Y\}$ (since the function governing the decision weights can vary across k), but says that if the person is initially indifferent between X and Y, then the addition of Z would lead him to choose X: Since the addition of Z brings up the average on the salary dimension and brings down the average on the days off dimension, condition (TC) tells us that it leads the person to care less about salary relative to days off, thereby making X look more attractive than Y. Cunningham's average-based formulation yields opposite predictions to our range-based formulation when, like in this example, adding an option impacts averages and ranges in different directions.

We can re-frame this example slightly to illustrate another point of comparison. Suppose that a person frames the jobs in terms of salary and vacation days, rather than salary and days off, where vacation days equal days off minus weekend days (with roughly 104 weekend days in a year). The idea is that the person's point of reference might be to be able to take off all weekend days rather than to take off no days. Then the problem can be re-written as choosing between the following jobs:

Job X. Salary: 100K, Vacation Days: 95
Job Y. Salary: 110K, Vacation Days: 85
Job Z. Salary: 120K, Vacation Days: 15,

where the person's underlying utility is represented by $U = \text{Salary} + 1000 \times \text{Vacation Days}$. This change in formulation does not influence the predictions of our model, or of Kőszegi and Szeidl's (2013), on how the person chooses from $\{X,Y\}$ or from $\{X,Y,Z\}$ because this change does not affect utility ranges along the different dimensions. On the other hand, this change does influence the predictions of Bordalo, Gennaioli, and Shleifer (2013). Specifically, it alters the prediction of which choice the person makes from $\{X,Y\}$ because diminishing sensitivity is defined relative

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[54]As Bordalo, Gennaioli and Shleifer (2013) note, their model accommodates the attraction effect when people choose between options that vary in quality and price. For example, people will choose $(70,-20)$ from $\{(70,-20),(80,-30)\}$ but will instead choose $(80,-30)$ from $\{(70,-20),(80,-30),(80,-40)\}$ since the addition of the decoy option $(80,-40)$ makes the price of the middle option not salient because it equals the average price. However, their model also robustly accommodates the opposite effect — call it the "repulsion effect" — in these situations. In particular, the person will choose $(70,-20)$ if the price of decoy is made larger so price becomes salient for the middle option. For example, the person will choose $(70,-20)$ from $\{(70,-20),(80,-30),(80,-70)\}$.

to a reference point: With the new reference point, vacation days are now more rather than less salient for both options in $\{X,Y\}$ because a 5K difference looms larger relative to an average of 90K than 105K, implying that a person chooses X rather than Y from the binary choice set.[55] And while this particular change in the reference point does not alter the qualitative predictions of Cunningham (2013), a different change does: Suppose a person uses a reference point where all 365 days are taken off and each option is represented in terms of (Salary, Work Days), where utility is represented by $U = \text{Salary} - 1000 \times \text{Work Days}$. In this case, Cunningham (2013) says that the addition of Z reduces the person's sensitivity to work days, since it raises the average number of such days, while the earlier framing in terms of days off instead suggested that the addition of Z would increase the person's sensitivity to work days since it decreased the average number of days off.

D   Proofs

Proof of Proposition 1. Let $d(c',c|C) \in \mathbb{R}^K$ denote a vector that encodes proportional differences with respect to the range of consumption utility: For all k,

$$d_k(c',c|C) = \frac{\delta_k(c',c)}{\Delta_k(C)}.$$

We have

$$U^N(c|C) - U^N(c'|C) = \sum_j w(\Delta_j(C)) \left[u_j(c_j) - u_j(c'_j) \right] = \sum_j w\left(\frac{\delta_j(c,c')}{d_j(c,c'|C)} \right) \delta_j(c,c') \geq 0, \quad (4)$$

where the inequality follows from the person being willing to choose c from C.

For part 1, suppose $\tilde{C}$ is a k-widening of C with $\delta_k(\tilde{c},\tilde{c}') > \delta_k(c,c') > 0$, $d_k(\tilde{c},\tilde{c}'|\tilde{C}) = d_k(c,c'|C)$, and $\delta_i(\tilde{c},\tilde{c}') = \delta_i(c,c') \ \forall i \neq k$. Then

$$U^N(\tilde{c}|\tilde{C}) - U^N(\tilde{c}'|\tilde{C}) = U^N(c|C) - U^N(c'|C) + \left[w\left(\frac{\delta_k(\tilde{c},\tilde{c}')}{d_k(c,c'|C)} \right) \delta_k(\tilde{c},\tilde{c}') - w\left(\frac{\delta_k(c,c')}{d_k(c,c'|C)} \right) \delta_k(c,c') \right]$$

$$> U^N(c|C) - U^N(c'|C) \text{ (by } N2\text{)},$$

so the person is not willing to choose $\tilde{c}'$ from $\tilde{C}$.

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[55] Specifically, given $C = \{X,Y\}$, vacation days are salient for X since $95/90 > 105/100$ and are salient for Y since $90/85 > 110/105$.

For part 2, suppose $\tilde{C}$ is a k-widening of C with $\delta_k(c,c') < 0$ and $\delta_i(\tilde{c},\tilde{c}') = \delta_i(c,c')\ \forall i$. Then

$$U^N(\tilde{c}|\tilde{C}) - U^N(\tilde{c}'|\tilde{C}) = U^N(c|C) - U^N(c'|C) + \left[w\left(\frac{\delta_k(c,c')}{d_k(\tilde{c},\tilde{c}'|C)} \right) \delta_k(c,c') - w\left(\frac{\delta_k(c,c')}{d_k(c,c'|C)} \right) \delta_k(c,c') \right]$$

$$> U^N(c|C) - U^N(c'|C) \text{ (by } N1\text{)},$$

so the person is not willing to choose $\tilde{c}'$ from $\tilde{C}$.

$\blacksquare$

Proof of Proposition 2. For the first part, suppose each c is measured in utility units. The result trivially holds whenever $K = 1$ or $K = 2$, since the person will always choose to maximize consumption utility from a binary choice set for such K, so suppose $K \geq 3$. Let $\bar{\Delta} = \max_j \Delta_j(\{c,c'\})$ and m be a value of j satisfying $\Delta_m = \bar{\Delta}$. Further, let $\bar{c}_i = \max\{c_i, c_i'\}$ and $\underline{c}_i = \min\{c_i, c_i'\}$. Now construct c'' as follows:

- $c_m'' = \underline{c}_m$

- $c_k'' = \underline{c}_k + \bar{\Delta}$ for some $k \neq m$

- $c_i'' = \bar{c}_i - \bar{\Delta}$ for all $i \neq k, m$.

Note that c'' is not (strictly) dominated by c or c' since $c_k'' \geq \bar{c}_k$.

Since $\Delta_j(C) = \bar{\Delta}$ for all j by construction, the agent will make a utility-maximizing choice from C. To complete the proof, we need to verify that this choice is in fact c', or $U(c'') \leq U(c')$:

$$\sum_i c_i'' = \underline{c}_m + \underline{c}_k + \bar{\Delta} + \sum_{i \neq k,m} (\bar{c}_i - \bar{\Delta})$$

$$\leq \sum_{i=1}^K \underline{c}_i + \bar{\Delta} \text{ (because } \bar{c}_i - \underline{c}_i \leq \bar{\Delta})$$

$$\leq \sum_i c_i'.$$

For the second part, first consider the "if" direction. Suppose (1) holds, and let the comparison set equal $\{c, c', c''\}$, where c'' is defined such that

$$u_j(c_j'') = \begin{cases} u_j(c_j') & \text{if } j \in A(c',c) \text{ or } j \in E(c',c) \\ -\bar{u} & \text{otherwise,} \end{cases}$$

where $\bar{u} > 0$ and $-\bar{u} < \min_k u_k(c_k')$.

For $C = \{c, c', c''\}$ we have that

$$U^N(c'|C) - U^N(c|C) = \sum_{i \in A(c',c)} w(\delta_i(c',c)) \cdot \delta_i(c',c) + \sum_{i \in D(c',c)} w(u_i(c_i) + \bar{u}) \cdot \delta_i(c',c)$$

$$\geq \sum_{i \in A(c',c)} w(\delta_i(c',c)) \cdot \delta_i(c',c) + w\left(\min_{k \in D(c',c)} u_k(c_k) + \bar{u}\right) \sum_{i \in D(c',c)} \delta_i(c',c),$$

which exceeds 0 for sufficiently large $\bar{u}$ by *N1* and (1). Since it is also true that $U^N(c'|C) - U^N(c''|C) = \sum_{i \in D(c'',c)} w(u_i(c_i) + \bar{u}) \cdot (u_i(c'_i) + \bar{u}) > 0$, the person chooses c' from $\{c, c', c''\}$ when $\bar{u}$ is sufficiently large. Note that, by continuity, this argument also goes through if c'' is slightly perturbed so as not to be dominated.

For the "only if" direction, suppose condition (1) does not hold. Then, for any C containing $\{c, c'\}$,

$$U^N(c'|C) - U^N(c|C) = \sum_{i \in A(c',c)} w(\Delta_i(C)) \cdot \delta_i(c',c) + \sum_{i \in D(c',c)} w(\Delta_i(C)) \cdot \delta_i(c',c)$$

$$< \sum_{i \in A(c',c)} w(\delta_i(c',c)) \cdot \delta_i(c',c) + \sum_{i \in D(c',c)} w(\infty) \cdot \delta_i(c',c)$$

$$\leq 0,$$

where the first inequality follows from *N1*.

∎

Proof of Corollary 1. Let $d(c',c|C) \in \mathbb{R}^K$ denote a vector that encodes proportional differences with respect to the range of consumption utility: For all k,

$$d_k(c',c|C) = \frac{\delta_k(c',c)}{\Delta_k(C)}.$$

1. If c dominates c', then $D(c',c)$ is non-empty, while $A(c',c)$ is empty, implying that condition (1) does not hold. The result then follows from Proposition 2.

2. Fix $\tilde{\delta}_A$. The left-hand side of condition (1) equals

$$\sum_{i \in A(c',c)} w(\delta_i(c',c)) \cdot \delta_i(c',c) - w(\infty) \cdot \delta_D(c',c) \leq \left\{ \sup_{\{\mathbf{d} \in \mathbb{R}_+^K : \sum_i d_i = \tilde{\delta}_A\}} \sum_{i=1}^K w(d_i) \cdot d_i \right\} - w(\infty) \cdot \delta_D(c',c).$$

Clearly, the right-hand side of the above inequality falls below 0 for $\delta_D(c',c)$ sufficiently large when *N3* holds. The result then follows from Proposition 2.

59

■

Lemma 1. *For all non-degenerate distributions F with support on $[x,y]$, $y > x$, we have*

$$[E[F] - 1/2 \cdot S(F), E[F] + 1/2 \cdot S(F)] \subset [x,y].$$

Proof. We have

$$\begin{aligned}
E[F] + 1/2 \cdot S(F) &= E[F] + 1/2 \cdot \int \int |c - c'| dF(c) dF(c') \\
&= E[F] + 1/2 \cdot \int \int 2\max\{c,c'\} - (c+c') dF(c) dF(c') \\
&= E[F] + 1/2 \cdot [2E_F[\max\{c,c'\}] - 2E[F]] \\
&= E_F[\max\{c,c'\}] \\
&< y \text{ (for non-degenerate } F).
\end{aligned}$$

We can similarly establish that $E[F] - 1/2 \cdot S(F) > x$ for non-degenerate F. ■

Remark 1. The proof of Lemma 1 establishes that $E[F] + 1/2 \cdot S(F) = E_F[\max\{c,c'\}]$, and we can similarly establish that $E[F] - 1/2 \cdot S(F) = E_F[\min\{c,c'\}]$. This provides an alternative expression for $\Delta_k(\mathscr{F})$:

$$\Delta_k(\mathscr{F}) = \max_{F \in \mathscr{F}} E_F[\max\{u_k(c_k), u_k(c'_k)\}] - \min_{F \in \mathscr{F}} E_F[\min\{u_k(c_k), u_k(c'_k)\}].$$

Proof of Proposition 3. It will be useful to recall Lemma 1 in Kőszegi and Rabin (2007): if F' is a mean-preserving spread of F and $F' \neq F$, then $S(F) < S(F')$.

For the first part of the proposition, let $\mathscr{F} = \{(F_1, F_2), (F_1 - G_1, F_2 + G_2)\}$ and $\mathscr{F}' = \{(F_1, F'_2), (F_1 - G_1, F'_2 + G'_2)\}$. Since (F_1, F_2) is chosen from $\mathscr{F}$, we have

$$U^N((F_1, F_2)|\mathscr{F}) - U^N((F_1 - G_1, F_2 + G_2)|\mathscr{F}) = w(\Delta_1(\mathscr{F})) \cdot E[G_1] - w(\Delta_2(\mathscr{F})) \cdot E[G_2] \geq 0, \tag{5}$$

where

$$\begin{aligned}
\Delta_1(\mathscr{F}) &= E[G_1] + \frac{1}{2}\left(S(F_1) + S(F_1 - G_1)\right) \\
\Delta_2(\mathscr{F}) &= E[G_2] + \frac{1}{2}\left(S(F_2 + G_2) + S(F_2)\right).
\end{aligned}$$

Since F'_2 is a mean-preserving spread of F_2 and G'_2 is a mean-preserving spread of G_2, we also have that $F'_2 + G'_2$ is a mean-preserving spread of $F_2 + G_2$, so Lemma 1 in Kőszegi and Rabin (2007) tells

60

us that $\Delta_2(\mathscr{F}') \geq \Delta_2(\mathscr{F})$ with strict inequality whenever $F_2' \neq F_2$ or $G_2' \neq G_2$. Since it is also the case that $\Delta_1(\mathscr{F}') = \Delta_1(\mathscr{F})$, Equation (5) then implies that $U^N((F_1,F_2')|\mathscr{F}') - U^N((F_1 - G_1, F_2' + G_2')|\mathscr{F}') \geq 0$ by *N1*, with strict inequality whenever $F_2' \neq F_2$ or $G_2' \neq G_2$.

It remains to show the second part of the proposition. Let $\mathscr{F}(G_1,G_2)$ denote the comparison set when the decision-maker faces the distribution over choice sets of the form $\{(0,0),(-\tilde{x},\tilde{y})\}$ that is induced by drawing $\tilde{x}$ from G_1 and $\tilde{y}$ independently from G_2, where $G_1 \in \{F_1,F_1'\}$ and $G_2 \in \{F_2,F_2'\}$.

The range on each dimension equals the range when we restrict attention to the subset of $\mathscr{F}(G_1,G_2)$ generated by the union of the lotteries associated with "always choose $(0,0)$" and "always choose $(-\tilde{x},\tilde{y})$". The first of these lotteries yields $E_F[u_k(c_k)] \pm \frac{1}{2}S_F[u_k(c_k)] = 0$ along each dimension, while the second yields $-E[G_1] \pm 1/2 \cdot S(G_1)$ along the first and $E[G_2] \pm 1/2 \cdot S(G_2)$ along the second dimension.

By Lemma 1, the range on the dimensions are then

$$\Delta_1(\mathscr{F}(G_1,G_2)) = E[G_1] + \frac{1}{2}S(G_1)$$

$$\Delta_2(\mathscr{F}(G_1,G_2)) = E[G_2] + \frac{1}{2}S(G_2).$$

Consequently, $\Delta_2(\mathscr{F}(F_1,F_2)) < \Delta_2(\mathscr{F}(F_1,F_2'))$ whenever (i) $F_2' \neq F_2$ is a mean-preserving spread of F_2, as, in this case, $E[F_2] = E[F_2']$ and $S(F_2') > S(F_2)$ by Lemma 1 in Kőszegi and Rabin (2007), or (ii) F_2' first order stochastically dominates F_2, as, in this case, $E[F_2'] + 1/2 \cdot S(F_2') = E_{F_2'}[\max\{\tilde{y},\tilde{y}'\}] > E_{F_2}[\max\{\tilde{y},\tilde{y}'\}] = E[F_2] + 1/2 \cdot S(F_2)$, where the equality comes from Remark 1 and the inequality is obvious. From (i) and (ii), $\Delta_2(\mathscr{F}(F_1,F_2)) < \Delta_2(\mathscr{F}(F_1,F_2'))$ whenever $F_2' \neq F_2$ first order stochastically dominates a mean-preserving spread of F_2.

The result then follows from the fact that

$$U^N((0,0)|\mathscr{F}) - U^N((-x,y)|\mathscr{F}) = w(\Delta_1) \cdot x - w(\Delta_2) \cdot y$$

is increasing in Δ_2 by *N1*. ∎

Proof of Proposition 4. First, consider the case where c'' spreads out the advantages of c' relative to c. In this case, there exists a $j \in E(c',c)$, $k \in A(c',c)$, and $\varepsilon < \delta_k(c',c)$ such that

$$U^N(c''|\{c,c''\}) - U^N(c|\{c,c''\}) = U^N(c'|\{c,c'\}) - U^N(c|\{c,c'\})$$
$$+ w(\delta_k - \varepsilon) \cdot (\delta_k - \varepsilon) + w(\varepsilon) \cdot \varepsilon - w(\delta_k) \cdot \delta_k.$$

Supposing that $\varepsilon \leq \frac{\delta_k(c',c)}{2}$ (the case where $\frac{\delta_k(c',c)}{2} < \varepsilon < \delta_k(c',c)$ is analogous), the result then

follows from the fact that

$$w(\delta_k - \varepsilon) \cdot (\delta_k - \varepsilon) + w(\varepsilon) \cdot \varepsilon - w(\delta_k) \cdot \delta_k \geq w(\delta_k - \varepsilon) \cdot \delta_k - w(\delta_k) \cdot \delta_k > 0,$$

by successive applications of *N1*.

Now consider the case where c'' integrates the disadvantages of c' relative to c. In this case, there exists $j, k \in D(c', c)$ such that $U^N(c'' \mid \{c, c''\}) - U^N(c \mid \{c, c''\})$ equals

$$U^N(c' \mid \{c, c'\}) - U^N(c \mid \{c, c'\}) + w\left(|\delta_j(c', c) + \delta_k(c', c)|\right) \cdot (\delta_j(c', c) + \delta_k(c', c))$$
$$- \left[w(|\delta_j(c', c)|) \cdot \delta_j(c', c) + w(|\delta_k(c', c)|) \cdot \delta_k(c', c)\right].$$

The result then follows from the fact that

$$w\left(|\delta_j(c', c) + \delta_k(c', c)|\right) \cdot (\delta_j(c', c) + \delta_k(c', c)) > w(|\delta_j(c', c)|) \cdot \delta_j(c', c) + w(|\delta_k(c', c)|) \cdot \delta_k(c', c),$$

by *N1* (recall that $\delta_i(c', c) < 0$ for $i = j, k$). ∎

Proof of Proposition 5. From Condition (1) of Proposition 2 we want to show that

$$\sum_{i \in A(c'', c)} w(\delta_i(c'', c)) \cdot \delta_i(c'', c) + \sum_{i \in D(c'', c)} w(\infty) \cdot \delta_i(c'', c) > 0.$$

Since there is a C containing $\{c, c'\}$ such that c' is chosen from C, Condition (1) must hold for c', c. Noting that $\sum_{i \in D(c'', c)} w(\infty) \cdot \delta_i(c'', c) = \sum_{i \in D(c', c)} w(\infty) \cdot \delta_i(c', c)$, it suffices to show that

$$\sum_{i \in A(c'', c)} w(\delta_i(c'', c)) \cdot \delta_i(c'', c) \geq \sum_{i \in A(c', c)} w(\delta_i(c', c)) \cdot \delta_i(c', c),$$

which can be shown via an argument analogous to the one we made in the proof of Proposition 4. ∎

Proof of Proposition 6. The result is trivial for $K = 1$, so suppose $K \geq 2$. Let $C_\varepsilon = \{c\} \cup \{c^1\} \cup \ldots \cup \{c^K\}$, where, for each $j \in \{1, \ldots, K\}$, define $c^j \in \mathbb{R}^K$ such that

$$u_i(c_i^j) = \begin{cases} u_i(c_i) & \text{for } i = j \\ u_i(c_i) - \bar{u} & \text{for all } i \neq j, \end{cases}$$

supposing $\bar{u}$ is sufficiently large that $w(\bar{u}) - w(\infty) < w(\infty) \cdot \varepsilon \equiv e$.

By construction, $\Delta_i(C_\varepsilon) = \bar{u}$ for all i, so the person makes a utility maximizing choice from C_ε, which is c.

Also, for any $\tilde{C}$ containing C_ε, $\Delta_i(\tilde{C}) \geq \Delta_i(C_\varepsilon)$, so $w(\Delta_i(\tilde{C})) < w(\infty) + e$ for all i. This means that, for any $c' \neq c \in \tilde{C}$, we have

$$U^N(c'|\tilde{C}) - U^N(c|\tilde{C}) = \sum_{i \in A(c',c)} w(\Delta_i(\tilde{C})) \cdot \delta_i(c',c) + \sum_{i \in D(c',c)} w(\Delta_i(\tilde{C})) \cdot \delta_i(c',c)$$
$$< (w(\infty) + e) \cdot \delta_A(c',c) - w(\infty) \cdot \delta_D(c',c),$$

where this last term is negative (meaning c' will not be chosen from $\tilde{C}$) whenever $\delta_A(c',c) = 0$ or $\delta_A(c',c) > 0$ and $\frac{\delta_D(c',c)}{\delta_A(c',c)} - 1 > \varepsilon$. ∎

Proof of Corollary 2. This result is trivial if $K = 1$ or $\delta_A(c',c) = 0$, so let $K \geq 2$ and $\delta_A(c',c) > 0$. Since $U(c) > U(c')$,

$$\lambda \equiv \frac{\delta_D(c',c)}{\delta_A(c',c)} - 1 > 0.$$

Let $C = \{c'\} \cup C_{\lambda'}$, where $C_{\lambda'}$ is constructed as in the proof of Proposition 6 (letting $\varepsilon = \lambda'$), with $\lambda' = \lambda - \eta$ for $\eta > 0$ small.

By Proposition 6, c would be chosen from $C_{\lambda'}$ and c' would not be chosen from any $\tilde{C}$ containing $C_{\lambda'}$ and c', including from C. It is left to establish that c would be chosen from C, but this follows from the fact that $U^N(c|C) - U^N(c^j|C) = \sum_{i \neq j} w(\Delta_i(C)) \cdot \bar{u} \geq 0$ for any $c^j \in C_{\lambda'}$. ∎